Why be a Height Potentialist?

Zeynep Soysal

Abstract: According to height potentialism, the height of the universe of sets is "potential" or "indefinitely extensible," and this is something that a (formal) theory of sets should capture. Height actualism is the rejection of height potentialism: the height of the universe of sets isn't potential or indefinitely extensible, and our standard non-modal theories of sets don't need to be supplemented with or reinterpreted in a modal language. In this paper, I examine and (mostly) criticize arguments for height potentialism. I first argue that arguments for height potentialism that appeal to its explanatory powers are unsuccessful. I then argue that the most promising argument for height potentialism involves the claim that height potentialism follows from our intuitive conception of sets. But, as I explain, on the most plausible way of developing this argument from an intuitive conception of sets, it turns out that whether height potentialism or height actualism is true is a verbal dispute, i.e., a matter of what meanings we choose to assign to our set-theoretic expressions. I explain that only pragmatic considerations can settle such a dispute and that these weigh in favor of actualism over potentialism. My discussion is also intended to serve two broader aims: to develop what I take to be the most promising line of argument for height potentialism, and to elaborate the height actualist position in greater detail than is standardly done.

Keywords: Actualism and potentialism in set theory, Analytic explanations, Analytic truths, The iterative conception of sets, The universe of sets

1. Introduction

According to *height potentialism* (henceforth simply 'potentialism'), the height of the universe of (pure) sets is "potential" or "indefinitely extensible." This is commonly taken to imply that a (formal or informal) theory that accurately describes the universe of sets should include modal statements that capture its potential or indefinitely extensible nature.[1] For example, potentialists usually claim that a theory that accurately describes the universe of sets should include the statement that any things can form a set (usually formalized in plural logic).[2] As I will understand it in this paper, *height actualism* (henceforth 'actualism') is the rejection of potentialism: according to actualism, the height of the universe of sets isn't potential or indefinitely extensible, and our standard non-modal theories of sets (which usually include at least part of Zermelo–Fraenkel set theory with the Axiom of Choice (henceforth 'ZFC')) don't need to be either supplemented with or reinterpreted in a modal language. Actualism, thus understood, is the default view that is at least implicit in the standard practice of set theory. Actualists can opt to go beyond rejecting potentialism and further characterize the universe of sets; one standard option for them is to characterize the universe of sets as a proper class, and another is to characterize it as a plurality that isn't singularized into an object (such as a set or a proper class).[3]

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[1] But not always: e.g., Bacon (2023) argues instead for a formalization of the extensibility of the universe of sets in higher-order logic.

[2] Those who accept or attribute that statement to potentialists include Linnebo (2010, 145), Linnebo (2013, 219), Studd (2013, 699), Menzel (2018, 301), Scambler (2021, 1081), Builes and Wilson (2022, 12), Roberts (unpublished, 4). Warren (2017, 82, 102) accepts similarly that we can always come to a more expansive understanding of our quantifiers, but doesn't formalize this statement in modal terms.

[3] Those that opt for the former include Drake (1974, 2), Kunen (1980, 23), Lewis (1991), Jech (2003, 5) those that opt for the latter include Boolos (1984), Burgess (2004), or Uzquiano (2003). Of note is that potentialists can also accept the existence of proper classes understood as intensional entities; for discussion, see, e.g., (Barton, 2024).

In this paper, I will examine and (mostly) criticize arguments for potentialism. My main contentions will be that the dispute between actualism and potentialism is best understood as verbal, that because of this, only pragmatic considerations can adjudicate between potentialism and actualism, and, finally, that such pragmatic considerations seem to favor actualism over potentialism. My discussion will also serve two broader aims, which are to develop what I take to be the most promising line of argument for potentialism and to elaborate the actualist position in greater detail than is standardly done.

There are two types of arguments standardly given for potentialism. The first, and most common type, are arguments that appeal to explanatory power. The idea is that potentialism is true because actualism leaves certain facts unexplained or arbitrary—such as the fact that there is no set of all sets, or that there can't be more sets than there actually are—whereas potentialism doesn't. I will argue that arguments from explanatory power for potentialism are unsuccessful. In particular, I will explain that either the potentialist's explanations of the relevant facts are no better than the actualist's, or there are only unexplained facts if potentialism is true to begin with.[4]

The second type of argument for potentialism appeals to what is part of our intuitive conception of sets. The idea is that our intuitive conception of sets has modal elements: for instance, as Cantor seems to have thought, there are multiplicities that are not "finished" or that "cannot exist together" (Cantor, 1899), or, as Zermelo seems to have thought, the universe of sets is "open-ended" (Zermelo, 1930). The argument from an intuitive conception of sets, then, is that the universe of sets is potential because this is part of the intuitive conception of sets that we are working with. Since an accurate (formal) theory of sets is supposed to capture our intuitive conception of sets, it should thus include modal statements. As I will explain, this second type of argument for potentialism is much more promising: if our intuitive conception of sets really has modal elements, then whatever story one has for thinking that an intuitive conception of sets is true can also be used to justify these modal elements. I will argue that on the most plausible story for thinking that an intuitive conception of sets is true, the intuitive conception of sets is meaning-determining, that is, the intuitive conception of sets implicitly defines the meanings of the primitive set-theoretic expressions in it. But then, since there are also clearly non-modal intuitive conceptions of sets (such as the actualist conception), the argument from an intuitive conception of sets, if sound, will entail that whether potentialism or actualism is true is a verbal dispute, that is, a matter of what meanings we choose to assign to our set-theoretic expressions. I will explain that only pragmatic considerations can settle such a dispute. And I will suggest that these pragmatic considerations weigh in favor of actualism over potentialism.

I will discuss arguments from explanatory power first (§2) and then turn to arguments from intuitive conception (§3).

2. Arguments from explanatory power

It is very common for potentialists to motivate potentialism by claiming that it has some kind of explanatory superiority over actualism. More specifically, potentialists argue that their view can explain facts that actualism leaves unexplained or arbitrary. I will set aside the question of why (and whether) having explanatory superiority is evidence for the truth of a mathematical theory until §3; here in §2, I will examine the claim that potentialism has explanatory superiority over actualism.

Potentialists have described three different (though related) facts that are purportedly unexplained or arbitrary on the actualist view: the first is that the universe of sets (just as certain other collections of sets) doesn't actually form a set, the second is that the universe of sets (just as certain other collections of sets) doesn't possibly form a set, and the third is that the universe of sets (just as certain other collections of sets) isn't possibly "higher" than it actually is. I will discuss each in turn.

2.1. Unexplained actual non-existence

As is well-known, standard set theory precludes the existence of the sets of all and only the sets to which certain conditions apply, such as the condition of being non-self-membered, being an ordinal number, or being

[4] Some of these arguments expand upon my work in (Soysal, 2017a, 2017b).

a cardinal number. Potentialists commonly remark that the actualist view leaves it unexplained why these sets don't exist; or, as James Studd puts it, it leaves unanswered the question: "*What is it about the world that allows some sets to form a set, whilst prohibiting others from doing the same?*" (Studd, 2013, 699). See, for instance, Øystein Linnebo:

> According to the actualist conception, the set-theoretic quantifers range over a definite totality of all sets. Why should the objects that make up this totality not themselves form a set? Since a set is completely specified by its elements, we can give a precise and complete specification of the set that these objects would form if they did form a set. What more could be needed for such a set to exist? (Linnebo, 2013, 206)[5]

Potentialists usually go on to present potentialism as filling this explanatory gap and thus as having an explanatory advantage over actualism.[6] The first argument from explanatory power for potentialism, then, is that actualism leaves it unexplained why there is no set of all non-self-membered sets (or ordinals, or cardinals, and so on), while potentialism doesn't.

Consider, first, the claim that actualism leaves these non-existence facts unexplained. An obvious reply for the actualist is to say that what explains that there is no set of all non-self-membered sets (or ordinals, or cardinals, and so on) is that it is contradictory to assume that it exists. After all, the standard textbook explanations for why there are no such sets involve mathematical arguments that show that their existence is contradictory. Take, for instance, the condition of being non-self-membered. The following reasoning, known as 'Russell's Paradox', shows that there is no set that contains all and only the non-self-membered sets: Assume that there is such a set. Call it 'r'. Since r contains all and only the sets that are non-self-membered, r contains r if and only if r is non-self-membered. That is, r contains r if and only if r doesn't contain r. This is a contradiction. Similarly, the Burali–Forti Paradox and Cantor's Paradox yield arguments against the existence of the set of all and only the ordinals and cardinals, respectively. I will call the explanations given by these mathematical arguments 'textbook explanations'.

Potentialists commonly remark that these textbook explanations that are available to the actualists don't explain the non-existence of the relevant sets. See, for instance, James Studd:

> The derivation of Russell's paradox in Naïve Set Theory demonstrates the logical falsity of the instance of the Naïve Comprehension schema [. . .]. This provides—I am happy to grant—as good an explanation as we should expect for why this theory is inconsistent. However, the question of real interest is not why this instance of Naïve Comprehension yields a contradiction, but why certain sets—in this case, those that lack themselves as elements—are unable to form a set. And this cannot be explained merely by appeal to logical truths. (Studd, 2013, 700)

Or see Michael Dummett in this often-quoted passage:

> A mere prohibition leaves the matter a mystery. [. . .] And merely to say, "If you persist in talking about the number of all cardinal numbers, you will run into contradiction," is to wield the big stick, but not to offer an explanation. (Dummett, 1991, 315f.)[7]

It is difficult to see what the problem with the textbook explanations is. It would be very implausible to deny that mathematical arguments (that is, proofs) in general can explain mathematical facts. It would be similarly implausible to deny that proofs by contradiction can explain mathematical facts; even constructivist

[5] See also, e.g., "[. . .] why does the hierarchy itself fail to be completed so as to constitute a set? [. . .] Why do all the urelements and sets that there actually are fail to constitute a further level that kicks off yet another series of iterations?" (Menzel, 2018, 298).

[6] See, e.g., (Parsons, 1977, 345f.), (Linnebo, 2013, 206), (Yablo, 2006, 146, 152–155), or (Barton, 2023, 9f.).

[7] See also Yablo (2006, 149). Scambler also seems to think that there is something unexplained or paradoxical about the fact that certain collections don't (actually) form a set: "we are left with some things that, potentially 'paradoxically', *are not* and cannot in principle be collected together into a set" (Scambler 2021, 1081, *my emphasis*).. Or see also Glanzberg: "But of course, to the extent that our notion of set is well-defined, V looks like a perfectly good set. We can insist it is not, but we lack a good explanation of why not" (Glanzberg, 2021, 1493).

mathematicians accept proofs by contradiction that establish the negation of some mathematical claim, including the negation of an existential claim. In a sense, the fact that the existence of a set of all non-self-membered sets is contradictory is the best explanation we could expect for why there is no such set; what more could we hope for as an explanation for why something doesn't exist than a proof that its existence is logically impossible?

As I will explain in §3.2, my view is that the textbook explanations are perfectly good explanations because they are *analytic explanations*, that is, they are arguments that show that the explanandum is entailed by *analytic truths*, or truths that follow from the implicit definitions of some relevant expressions. More specifically, my view is that the existence of the set of all sets (or ordinals, or cardinals, and so on) contradicts principles that follow from the implicit definitions of our set-theoretic expressions. As I see it, then, the textbook explanations only appear problematic if one doesn't realize why the principles that contradict the existence of the sets in question are true to begin with; in these cases, the textbook explanations appear to not be "deep" enough as explanations. Once one realizes that the principles in question are analytic, however, the textbook explanations become perfectly satisfactory.[8] As I will explain in §3.2, these (analytic) textbook explanations are available to both potentialists and actualists. But potentialists don't (currently) share my views about the textbook explanations. So, what are their complaints about the actualist's use of textbook explanations?

One can find an articulation of the potentialist's worry about the textbook explanations in (Linnebo, 2018, 56f.).[9] Linnebo seems to grant that proofs (by contradiction) can explain mathematical facts—even the fact that there is no set of all non-self-membered sets (or all ordinals, cardinals, and so on). However, according to him, such explanations only go through on an "intensional" specification of the sets in question, and not on an "extensional" one:

> Consider the collection of objects that are not elements of themselves. Can we form a set whose elements are the members of this collection? [...] [T]he answer depends on how the target collection is specified. Suppose the target is specified intensionally, say, by means of the condition $x \notin x$. Then the characterization of the desired set is logically incoherent. We want a set r such that $\forall x(x \in r \leftrightarrow x \notin x)$, but a simple theorem of first-order logic tell us that there is no such thing. Suppose instead that the target collection is specified in a purely extensional way, say, as a (perhaps infinite) list that includes each and every non-self-membered set in our domain. Then there is no logical or conceptual obstacle to the formation of a set whose elements are precisely the items on this list. All that the paradoxical reasoning shows is that the set cannot itself be on the list and in this sense has to be "new". But this conclusion is unproblematic. There is no logical or conceptual obstacle to the formation of "new" objects beyond those that figure on some given list. (Linnebo, 2018, 56f.)

According to Linnebo, a domain can be "extensionally specificed" when it can be "specified as a plurality of objects"; and if a domain can only be specified "intensionally" then it "doesn't admit of a specification as a plurality" (Linnebo, 2018, 53) Linnebo further specifies what an "extensional specification" is by saying that a list of the members of a collection is an extensional specification of the collection, but the order of the list doesn't matter:

> The linear order of the list is immaterial. Suppose we wish to keep track of some web pages to which a new web page is intended to link. Suppose we write each of the target URLs on a separate piece of paper. It does not matter if the pieces are rearranged. Together, the pieces still provide a purely extensional specification of the target collection. (Linnebo, 2018, 57)

On this reading, then, the argument from explanatory power is that the actualist can't explain why the collection of all sets when it is specified extensionally doesn't form a set, while the potentialist can.

Consider once again the claim that the actualist leaves some non-existence fact unexplained. Whenever some collection of sets is "extensionally specified" via a list (either ordered or not), the actualist should say that this collection already (actually) forms a set. Indeed, any list of sets (certainly any finite list, but also any list of some cardinality) will already provably form a set for the actualist, for instance, via applications of the Axiom of Replacement.[10] Now, if lists (ordered or not) are the only way to specify a collection "extensionally,"

8 I have also argued for this in (Soysal, 2017a, 2017b).
9 See also (Linnebo and Shapiro, 2017, 179).

then the actualist should say that there is no extensional specification of the collection or domain of all sets on the actualist conception; the only way to specify or refer to the totality of all actually existing sets is by using expressions such as “all sets,” or “the totality of all actually existing sets,” or “everything to which my expression ‘set’ applies,” and so on. The actualist can then invoke a textbook explanation to explain why these sets thus specified don’t form a set. For instance, the actualist can say that no actually existing set in the totality of “all sets” is self-membered (a consequence of the Axiom of Foundation and the Axiom of Separation); therefore, there is no set that contains “all sets” in this totality of “all sets,” since otherwise it would have to be self-membered. (This is my favorite version of the textbook explanation, which I will flesh out further in §3.2.) Linnebo should find this explanation satisfactory, given that he grants that textbook explanations are explanatory when they concern intensionally specified collections.

Can a collection of sets be specified extensionally in a way other than via a list (ordered or not)? Linnebo seems to claim that, more generally, if one specifies a collection as a plurality, then one has specified it extensionally (Linnebo, 2018, 57).[11] If that is the case, then one option for the actualist would be to reply that the totality of all sets can be specified extensionally because it is a (mere) plurality; as I mentioned in §1, it is common for actualists who want to further specify the universe of sets to specify it as a plurality that doesn’t form a set. The actualist can then run the same textbook explanation as above, but applied to the plurality of all sets: there is no set of the plurality of all sets because such a set would have to be self-membered, which contradicts the fact (which follows from the Axiom of Foundation and the Axiom of Separation) that no set in the plurality of all sets is self-membered. The actualist thus doesn’t leave it unexplained why there is no set of all the actually existing sets—whether specified extensionally or intensionally. Moreover, her textbook explanation is, on the face of it, just as explanatory in either case (I will defend that they are explanatory further in §3.2).

Potentialists might take issue with this last textbook explanation, because they usually deny that the totality of all sets forms a plurality (at any world).[12] On the potentialist view, whenever there are some sets, there “can” be a set of these things. So, whenever there are some sets at a world, that isn’t the “totality” of all sets, since there are still sets that can be formed from the ones in that world. For the potentialist, the “totality” of all sets in question is thus the totality of all actual and merely possible sets. But these potentialist claims don’t give the actualist reason to reject that the collection of all sets is a mere plurality, for the actualist simply rejects the potentialist’s claim that there are merely possible sets. These potentialist claims thus also don’t raise any new explanatory problem for the actualist: The potentialist hasn’t pointed to a fact that is unexplained on the actualist view by denying that the totality of all actual and possible sets forms a plurality. At best, the potentialist could object that the actualist leaves it unexplained why there aren’t merely possible sets, which is the explanatory challenge I consider in §2.2.

To conclude my discussion of the first argument from explanatory power, let me turn to its second claim, namely, that potentialism can explain the actual non-existence claims purportedly left unexplained on the actualist view. Here, some care is needed to specify what exactly is the explanandum that the potentialist is concerned with; as Sam Roberts (2023, fn. 20) has also pointed out, potentialists seem to subtly shift the question at issue. Recall that the actualist is accused of leaving unexplained why there is no set of all sets on her actualist picture. For the actualist, there are no merely possible sets, so the “universe” or “totality” of all sets refers to all the actually existing sets. The corresponding question that the potentialist should be answering, then, is: Why is there actually no set of all actually existing set on the potentialist picture?[13] For the potentialist, the set of all actually existing sets isn’t actual but merely possible. So, to understand why, on the potentialist

10 Presumably, a specification of all sets using some formula $\phi(x_1,...,x_n)$ of the language of set theory wouldn’t count as “extensional” for Linnebo, but otherwise the actualist could say that again provably by an instance of the (first-order) Reflection Principle, any collection thus specified already forms a set.

11 More specifically, (Linnebo, 2018, 57) says that a “simple but useful way to talk about extensionally specified collections is provided by the logic of plurals.” Arguably, this doesn’t give an account of when a collection can be specified extensionally, but only entails that we can refer to extensionally specified collections using plural language. But I consider this option here because it seems plausible that there should be an alternative to the account of extensionally specified collections as lists (ordered or unordered), and I take the account that says that extensionally specified collections are pluralities to be a plausible alternative that is also in the spirit of the text.

12 See, e.g., (Linnebo, 2010, 156–158), (Linnebo, 2013, 219f.).

13 Roberts (2023, fn. 20) calls this ‘the problem of actuality’, and argues that potentialists don’t have a clear response to it. This problem also seems implicit in Berry’s (2022, ch. 5) critical discussion of Parsonian potentialism and is related to a criticism of potentialism raised by Menzel (2018, 303f.). As I explain next, I think that the potentialist does have an explanation, but it is a bad one.

view, such a set is merely possible and not actual, we need to get a fix on the notion of "possibility" that the potentialist employs.

Potentialists have proposed different interpretations of their modality: one on which the modality is primitive and idiosyncratic to mathematics or set theory;[14] a second, "constructivist" interpretation on which sets don't exist timelessly and metaphysically necessarily, but rather come into existence via some kind of social process;[15] and a third, broadly "linguistic" interpretation on which the extensions of our expressions become more inclusive.[16]

It is clear that the potentialist doesn't have a satisfactory explanation on the first interpretive option: saying that the universe of sets doesn't actually form a set because it merely possibly forms one on a primitive sense of 'possibly' isn't an illuminating explanation, and certainly not for those who reject potentialism to begin with. But this point will become clearer in §3.1 where I explain the most plausible way of making sense of a primitive notion of modality for the potentialist, so I won't say more about it here.

On the constructivist interpretation, the existence of (pure) sets is metaphysically contingent; some type of social process brings about their existence at a point in time. On this interpretation, then, the potentialist's explanation for why there is no set of all actually existing sets is that this set hasn't yet been constructed or brought into existence by the relevant social process. The problem with this explanation is that it is extremely controversial to deny that abstract mathematical objects exist timelessly and necessarily (if they exist); even potentialists usually don't want to deny this.[17] It is arguably part of our intuitive notion of metaphysical possibility and of abstract mathematical objects that they exist timelessly and necessarily. Denying this is, at the very least, an important intuitive cost. Moreover, the view that sets are constructed by a social process in time is standardly taken to entail a weaker mathematical theory than ZFC.[18] For these reasons, and for anyone who endorses the standard view that mathematical objects exist timelessly and necessarily if they exist—and, certainly, for the actualist—the potentialist explanation on the constructivist interpretation will be highly unsatisfactory.

On the linguistic interpretation, the potentialist points out that we can (in the sense that it is physically possible for us to) change the extensions of our set-theoretic expressions (perhaps together with the range of the quantifiers) so that they apply to more things than they did before the change. As Timothy Williamson puts it:

> For given any reasonable assignment of meaning to the word 'set' we can assign it a more inclusive meaning while feeling that we are going on in the same way. (Williamson, 1998, 20)

Similarly, Linnebo (2018) explains that the modal operators "describe how the interpretation of the language can be shifted—and the domain expanded—as a result of abstraction," where "the intended meaning of '$\Box\varphi$' is 'no matter how we abstract and thereby shift the meaning of the language, φ'." (Linnebo, 2018, 205). On this third interpretation, then, the potentialist's explanation for why there is no set of all actually existing sets seems to be that this plurality isn't a set because we haven't yet shifted the meaning of our set-theoretic expressions (for instance, via abstraction) so as to call this mere plurality a 'set'. The problem with this explanation is that it doesn't address why there is no set of all actually existing sets on the actual usage of 'set'. The actualist can certainly grant the potentialist that we can change the meaning of 'set' so that it also applies to what the actualist calls 'proper classes' or 'mere pluralities'. But this doesn't address why those things the actualist calls 'proper classes' or 'mere pluralities' in the current, actual language don't form sets on the current, actual meaning of 'set'—which is precisely the why-question the potentialist was supposed to be answering. Like the actualist, the potentialist could respond that the reason why such collections aren't sets on the current, actual meaning of 'set' is given by some textbook explanation: their being a set would be contradictory, or perhaps logically incompatible with some basic facts about the application conditions of 'set', such as the fact that 'set' doesn't

14 Those who propose an idiosyncratic modality include Parsons (1977) and Linnebo (2010).

15 E.g., Linnebo and Shapiro (2018, 285) and Berry (2022, 60) discuss that interpretation of the potentialist's modality (though they don't endorse it).

16 See, e.g., Williamson (1998), Uzquiano (2015), Warren (2017), Linnebo (2018), Studd (2019).

17 See, e.g., (Parsons, 1977), (Linnebo, 2010, 158), (Studd, 2013, 706), (Linnebo and Shapiro, 2017, 167), (Linnebo and Shapiro, 2018, 288), (Studd, 2019, 146), (Warren, 2017, 113).

18 For discussion, see, e.g., (Linnebo and Shapiro, 2018, 285). For development of the argument, see, e.g., (Incurvati, 2012).

apply to anything that is self-membered (as I will further explain in §3.2). But the linguistic interpretation doesn't provide the potentialist with any alternative explanation for why there is no set of all actually existing sets—holding fixed the current meaning of 'set'.

I don't see how else potentialists can explain the actual non-existence claims.[19] Some potentialists are explicit that potentialism doesn't explain why there is no set of all non-self-membered sets (or of all actually existing sets, ordinals, cardinals, and so on), and even seem to grant that textbook explanations are satisfactory after all:

> [A]lthough the Russell reasoning shows there necessarily are some things that *aren't* the members of a set, nevertheless any possible things *can be* the members of a set: there are no 'special' things (plural) that somehow by their very nature cannot be the members of a set. (Scambler, 2021, 1081)

> Recall that RP [(Russell's Paradox)] shows that, necessarily, the plurality of all sets does not *actually* form a set. This leaves open, however, whether that plurality *could* form a set. [...] In response to RP, the modal strategy maintains that *within* any given set-theoretic structure certain pluralities will not form sets, but every such plurality *could* form a set in some larger set-theoretic structure. (Builes and Wilson, 2022, 12f.)[20]

When it comes to explaining the actual non-existence claims, then, the potentialist doesn't have an advantage over the actualist. At best, if the potentialist avails herself of some textbook explanation (as I argue in §3.2 that she should), then the actualist and the potentialist explanations are on a par.

As I mentioned above, at times it seems that the potentialist is concerned with a subtly different explanandum, namely, why the universe of sets *understood in the potentialist's sense* doesn't actually (or possibly) form a set. Charles Parsons, for instance, motivates the explanatory advantage of potentialism by explaining that when some multiplicity doesn't determine a set, "this is due to the fact that in a certain sense the multiplicity does not exist" (Parsons, 1977, 345):

> I suggest interpreting Cantor by means of a modal language with quantifiers, where within a modal operator a quantifier always ranges over a set [...]. Then it is not possible that all elements of, say, Russell's class exist, although for any element, it is possible that *it* exists. (Parsons, 1977, 346)

Linnebo motivates potentialism's explanatory advantage similarly in (Linnebo, 2013):

> On [the potentialist's] [...] conception, the hierarchy is potential in character and thus intrinsically different from sets, each of which is completed and thus actual rather than potential. This intrinsic difference affords potentialists [...] a reason to disallow the disputed set formation. (Linnebo, 2013, 206)

On one natural readings, Parsons and Linnebo here aren't concerned with explaining why there is no set of all actually existing sets, but, rather, with explaining why the totality of sets doesn't (and couldn't) form a set, where the "totality" of sets is understood as consisting of all actual and merely possible sets. The latter explanation, which we discussed earlier in §2.1, is that whenever there are some sets at a world, that isn't "the

[19] Perhaps potentialists could say that there is no set of all non-self-membered sets because the Axiom of Foundation is true (that is, every nonempty set is disjoint from one of its elements) and that they can explain why the Axiom of Foundation is true better than competitors; Linnebo (2013, 216f.), e.g., gives an argument for the Axiom of Foundation from a potentialist perspective. The problem with this argument is that there is no reason to think the potentialist's explanation of the Axiom of Foundation is any better than the actualist's, since the actualist can appeal to the iterative conception of sets, and it is well-known that the latter motivates the Axiom of Foundation perfectly well (the Axiom of Foundation is even considered essential to the iterative conception of sets). See also discussion in §3.

[20] Menzel says something similar: 'Note the question is not: Why is there no universal set, that is, no set containing all the urelements and all the sets? As we've just seen, the iterative conception of set provides a cogent answer to that question: only those pluralities that "run out" by some level of the cumulative hierarchy constitute sets at the next level and, obviously, the entire hierarchy is not such a plurality; there is no level at which the members of all the levels form a set' (Menzel, 2018, 298). After the quote I gave above, Studd says: "The derivation of a contradiction from the relevant instance of Naïve Comprehension shows that these sets do not form a set, but fails to explain why they *cannot*" (Studd, 2013, 700), which suggests that it might be better to interpret him also as granting that Russell's Paradox explains why there is no set of all sets, but as failing to explain why there couldn't be a set of all sets, as I discuss in §2.2.

totality of sets," because there are still sets that can be formed from the ones in that world; since the totality of sets doesn't exist at any given world, it also can't determine a set at any given world.

As I already explained, on this reading, the potentialist isn't raising an explanatory problem for the actualist. The actualist can explain why the universe of sets isn't actually a set, where "the universe of sets" is understood as the totality of all actually existing sets. Given that she doesn't recognize the need to introduce modal notions into set theory, she doesn't need to explain why "the universe of sets" understood as consisting of actual and merely possible sets doesn't form a set. In other words, the fact that the actualist doesn't explain why there is no set of all actual and possible sets doesn't show that there is something unexplained or arbitrary *on the actualist view* of the universe of sets, and thus it doesn't show that potentialism has an explanatory advantage. To take a somewhat facetious analogy, this would be like arguing for the existence of God by saying that it best explains why angels can transcend physical boundaries; an atheist isn't going to accept that there are angels to begin with. That being said, and again as I mentioned earlier, the potentialist could object that the actualist leaves it unexplained why there are no merely possible sets (why are there no angels?). This is the explanatory challenge to which I turn next.

2.2. Unexplained impossible existence

On the face of it, actualists reject that there can be more sets than there actually are. On a second reading, the argument from explanatory power for potentialism is that this is an unexplained fact on the actualist view: actualism leaves it unexplained why the universe of sets (as well as some other collections) *can't* form a set. This argument is implicit in the penultimate set of quotes from §2.1 that grant that standard mathematical arguments can explain why the problematic collections don't actually form a set, but point out that these arguments don't explain why these collections *couldn't* form sets. See, also, for instance:

> [W]ith the combinatorial/iterative conception in mind, why *can't* we "collect together" or "lasso" all the sets in the ZF hierarchy, and form the collection of them all? (Simmons, 2000, 111, *my emphasis*)

> [T]he Actualist takes there to be some plurality of objects (the sets) forming an iterative hierarchy structure [. . .]. But the following modal intuition seems appealing: for any plurality of objects satisfying the conception of an iterative hierarchy [. . .] *it would be in some sense (e.g., conceptually, logically or combinatorically if not metaphysically) possible* for there to be a strictly larger model of [a full-width iterative hierarchy] [. . .] which, in effect, adds a new stage above all the ordinals within the original structure together with a corresponding layer of classes. (Berry, 2022, 15, *my emphasis*)[21]

On the potentialist view, in contrast, these collections and pluralities *can* form sets; on the face of it, then, the potentialist view doesn't have unexplained facts about impossible set existence.

The key question for this second reading of the potentialist argument from explanatory power is: Does the actualist need to recognize that there are such modal facts in need of explanation? Once again, for the actualist, the universe of sets doesn't have a potential or modal nature. She follows the standard mathematical practice of interpreting 'can'-statements in terms of (actual) existence.[22] So, for instance, the actualist will say that on her view, there "can't" be a set of all sets in the sense that there *isn't* one, and there isn't one because the assumption that there is one entails a contradiction.

What the potentialist needs is to point to certain modal facts that the actualist needs to recognize, and ask why these facts don't hold on the actualist picture (if they don't). To do this, the potentialist needs to

[21] See also, e.g., Uzquiano who claims that we need "some independent reason to doubt that we *can* collect the non-self-membered sets in the totality into a set" (Uzquiano, 2015, 147, *my emphasis*).

[22] Linnebo and Shapiro (2018, 282) discuss (and concede) this standard practice to interpret 'can'-statements in mathematics non-modally. An alternative interpretation of 'can' is as epistemic possibility, i.e., there can't be a set of all non-self-membered sets in the sense that we know that there isn't one (because, e.g., we have a proof).

give an interpretation of her modality—and thus of the sense in which sets "can" or "can't" form sets—that is comprehensible to the actualist. Consider again the three standard potentialist interpretations of their modality.

It should seem clear that the actualist doesn't need to accept or explain that their universe of sets "can't" form a set in a primitive sense proper to the potentialist's explication of their potential hierarchy. But this point will become clearer after my discussion in §3.3, so I won't say more about it for now.

Recall that on the constructive interpretation, there "can" be more sets than there actually are in the sense that it is metaphysically possible for these (actually non-existing) sets to exist, as a result of some kind of social process of set construction. The challenge here, then, is that the actualist leaves it unexplained why it isn't metaphysically possible for there to be more sets than there actually are as a result of some process of social construction. The actualist can accept that there can't be more sets than there actually are in this sense, and she can give the standard explanation of that fact: there can't be more sets than there actually are in the constructivist sense because abstract mathematical objects exist timelessly and necessarily (if they exist), as is almost universally accepted (as we discussed in §2.1). On this second interpretation, there is as little unexplained on the actualist view as there is on any view which entails that abstract mathematical objects exist timelessly and necessarily, if they exist.

On the linguistic interpretation, the potentialist's challenge to actualism is that the actualist view leaves it unexplained why we (physically) can't assign more inclusive meanings or extensions to our set-theoretic expressions or quantifiers. The problem with this challenge is that the actualist can simply accept that we can assign more inclusive meanings and extensions to our set-theoretic expressions and quantifiers; she can even give the metasemantic explanation of that fact as the potentialists do.[23] So, it isn't an unexplained fact (or even a fact) on the actualist view that we can't assign more inclusive meanings or extensions to our set-theoretic expressions and quantifiers. The actualist can accept that we can assign more inclusive meanings, but she will just refuse to do so: For the actualist, 'set' refers to all and only the things that appear at some stage of the iterative hierarchy (which is given by the familiar intuitive story, perhaps together with the axioms of ZFC). If she wants, the actualist can further say that the "totality" of all sets, called 'V', is a proper class or a mere plurality. Either way, V isn't a set, on pain of contradiction. The actualist can grant that one could decide to use 'set' in a new and different way so as to call everything the actualist calls 'sets' *and* V a 'set'. But that term 'set' has a different extension (and thus also a different meaning), than the actualist's word 'set'; and this is something that the potentialist accepts. So, the actualist can agree that 'V can be a set' is true in the sense that, if we were to change the meaning of 'set' to refer to both sets and to V, then V would be a set. But the actualist doesn't want to change the meanings of her expression 'set'. For her, it would be best to disambiguate and use, for instance, 'shmet', to refer to the more inclusive extension. Then, when the potentialist asks: "But why can't the plurality you call 'V' form a set?" the actualist will say that V is a shmet, proper class, or mere plurality, but that it isn't a set given what she means by 'set'. Shmets, proper classes, and mere pluralities are similar to sets in some respects, but they aren't sets. And whoever adopts the more inclusive meaning of 'set' herself agrees with this: she agrees that there are distinct meanings to 'set' before and after the shift in meaning, and so that "new sets" and "old sets" aren't the same things.[24] The potentialist just insists on using 'set' with a new meaning instead of using 'shmet' or 'proper class' or 'mere plurality'. On this third interpretation, then, is no fact—let along an unexplained fact—that V can't form a set on the actualist picture; there are just different decisions about how to use set-theoretic expressions. In §3.4, I will explain that there are pragmatic reasons to prefer the actualist's use of 'set'. My point here is simply that there is no unexplained fact that sets can't form more sets on the actualist picture.[25]

2.3. Arbitrary height

Finally, potentialists also commonly object that the actualist is committed to a theory on which the "height" of the universe of sets is arbitrary. See, for instance:

[23] As will become clear in §3.4.2, actualists and potentialists actually accept a similar metasemantic theory; they just disagree on the actual use of the set-theoretic expressions and quantifiers (see footnote 39).

[24] See, e.g., the way Linnebo (2018, 55–59) explains this view.

[25] Linnebo (2018, 214–216) and Studd (2013, 711) argue that the potentialist can also explain which collections or pluralities can form sets and which cannot. From my arguments in this section (§2.2), it follows that, depending on the interpretation of the modality at issue, the actualist should either reject that these collections or pluralities can form sets or give the same explanation as the potentialist for why these (and only these) can form sets.

> Why is the hierarchy only as "high" as it is? Why do all the urelements and sets that there actually are fail to constitute a further level that kicks off yet another series of iterations? (Menzel, 2018, 298)

> The main challenge will be to motivate and defend the threshold cardinality beginning at which pluralities are too large to form sets. Why should this particular cardinality mark the threshold? Why not some other cardinality? (Linnebo, 2010, 152)[26]

There is a reading of these quotes on which the challenge for the actualist is to explain why it isn't *possible* for there to be more sets, leading back to the argument discussed in §2.2.[27] I take the challenge that is more specifically about the "height" of the universe of sets to go as follows. The actualist (just like the potentialist) is committed to the universe of sets being in some sense "as high as possible": this idea is standardly included as part of the intuitive iterative conception of sets—for instance, in the Cantorian idea of "absolute infinity" (Cantor, 1899)—and in set theorists' practice of accepting the existence of larger and larger cardinals.[28] Standard set theory entails that x isn't a set if it can be put in one-to-one correspondence with V or the ordinals Ω.[29] But this suggests that there is some "threshold height"—the "height" of V or Ω—beyond which collections don't form sets. According to the potentialist, this doesn't fit with the intuitive idea that the universe of sets is as high as possible; see, for instance, Linnebo in the passage following the above:

> Wherever it has been possible to go on to define larger sets, set theorists have in fact done so. So it remains arbitrary that there should be no sets of this [threshold] cardinality or some even larger one. (Linnebo, 2010, 153)

The third explanatory challenge for the actualist, then, is to explain why the universe of sets has this threshold height given that it should be as high as possible.

As I have previously argued (Soysal, 2017a, 2017b), I think the actualist can straightforwardly maintain both that the collection of all sets (or of all ordinals, or all cardinals) has a threshold "height" and that it is as "high" as possible.[30] Consider, first, that there is a specific notion of "height" (or "size") given in set theory by the notion of an ordinal (or cardinal): In set theory, ordinals (and cardinals) are defined as *sets* of a certain type.[31] On the standard definition, an ordinal β is *larger* than an ordinal α just in case $\alpha \in \beta$. With these specific explications of the informal notion of "height," "size," and "larger than," the actualist can then note that standard set theory entails that for every ordinal (cardinal) there is a larger one. Furthermore, the actualist can adopt the set-theoretic practice of accepting the existence of stronger and stronger theories that postulate the existence of larger and larger cardinals in this sense.[32] For the actualist, these facts suffice to capture the intuitive idea that the universe of sets is as "high" as possible.

The actualist will then think of the "height" or "size" of V differently. For one, the "height" or "size" of V isn't an ordinal or a cardinal (on pain of contradiction). The actualist could define a notion of "size" that applies to V; for instance, by saying that two classes have the same *shardinality* if and only if they can be put in one-to-one correspondence. Shardinality and cardinality are then both "sizes" in an informal sense, but they aren't the same type of "size," on pain of contradiction. For the actualist, not every type of "size" is a cardinality, again on pain of contradiction.

[26] See also, e.g., Yablo (2006, 152f., 155), Fine (2006, 23), Linnebo (2010, 152f.), Linnebo (2013, 206), Studd (2013, 700), Berry (2022, 14–17), Bacon (2023, 6f.).

[27] For instance, this is a natural reading of the way Berry (2022, 14–17) puts the challenge about height: " [T]he Actualist takes there to be some plurality of objects (the sets) forming an iterative hierarchy structure [. . .]. But the following modal intuition seems appealing: for any plurality of objects satisfying the conception of an iterative hierarchy [. . .] it would be in some sense (e.g., conceptually, logically or combinatorically if not metaphysically) possible for there to be a strictly larger model of [a full-width iterative hierarchy]" Berry (2022, 15).

[28] See, e.g., (Incurvati, 2017) for discussion of the role of such "maximality principles" in set theory.

[29] This is the Limitation of Size Principle, usually proved in a set theory with classes such as Neumann–Bernays–Gödel set theory (NBG). Linnebo (2010, 162) also derives it in set theory with pluralities.

[30] Burgess (2022) also briefly makes similar points against the potentialist's arbitrary height argument.

[31] Specifically, ordinals are sets that are transitive and well-ordered by $\in$, and cardinals are ordinals that aren't in one-to-one correspondence with any smaller ordinal.

[32] The practice described, e.g., in (Koellner, 2011).

These claims nicely parallel what the actualist says about "collections," as we saw at the end of §2.2: The universe V isn't a set (on pain of contradiction). The actualist can define a notion of "collection" that applies to it, for instance, by calling it a 'proper class' and defining these in some standard way. Then proper classes and sets are both "collections" in a loose sense, but they aren't the same type of "collection," on pain of contradiction. For the actualist, not every type of "collection" is a set, again on pain of contradiction.[33]

The actualist thus maintains both that the universe of sets is a "high" as possible (that is, more specifically, there is no largest cardinal/ordinal) and that there is a threshold "height" (that is, more specifically, a shardinal, or shordinal) above which "collections" (that is, more specifically, proper classes and sets, or mere pluralities and sets) don't form sets. This threshold isn't arbitrary: it is the first "height" at which the assumption that x has that height entails that x isn't a set. If she wants, the actualist could also accept that just as there are larger and larger ordinals and cardinals there are shlarger and shlarger shardinals and shordinals, by accepting the existence of collections that are neither sets nor proper classes, such as Super-Classes, Super-Super-Classes, and so on.[34] This would allow her to maintain the intuitive idea that there is no largest "size" in the loose sense, either. But, importantly, the idea that the universe of *sets* is as "large" as possible is captured by a more specific notion of "size" that applies to sets and that is defined in set theory. And once again, not all things that intuitively or informally count as "sizes" or "collections" are set-theoretic entities, on pain of contradiction; this is simply one of the fundamental commitments of the actualist view.

I will flesh out the actualist position and specifically the textbook "on pain of contradiction" explanation further in §3.2. But we can already conclude that the actualist can coherently maintain both that there is no arbitrary "stopping point" of the universe of sets and that there is a "size" in the loose sense above which collections are too "large" (in the loose sense) to form sets.

In the end, I don't think there is any hope for arguing for potentialism on the basis of its explanatory powers or advantages. This undermines a lot of what is standardly said in motivating potentialism. I do think that there is an alternative and more promising way to argue for potentialism. But, as we will see, this will also reveal that the debate over potentialism versus actualism isn't as substantial as one might have thought.

3. Arguments from an intuitive conception of sets

As I see it, a much more promising argument for potentialism starts from the observation that our intuitive conception of sets has some modal component. For some, the intuitive conception in question is the "iterative" conception of sets, which is sometimes taken to be implicit in the writings of Cantor and Zermelo. For instance, in the quote we saw earlier, Parsons suggests "interpreting Cantor by means of a modal language" (Parsons, 1977, 346), and, as the title of that paper suggests, he takes himself to be answering the question "What is the iterative conception of sets?" So, Parsons seems to think that the iterative conception of sets, implicit in the writings of Cantor, is best understood as having some modal component. Similarly, Linnebo (2013) starts by explaining that on the "familiar iterative conception" there "seems to be something inherently potential about the set theoretic hierarchy" (Linnebo, 2013, 205). Along the same lines, in their papers on potential infinity, Linnebo and Stewart Shapiro discuss how Cantor and Zermelo left room in their writings for some "limited form" of potential infinity (Linnebo and Shapiro, 2018, 286). It is also common for potentialists to focus on the naïve conception of sets and to suggest, for instance, that the modal statement that any sets can form a set "retains the intuitive plausibility of Naive Comprehension" (Builes and Wilson, 2022, 12).[35] The idea here seems to be that our intuitive conception of sets includes something like the claim that the extension of every predicate is a set, and that this is best understood as having a modal component. In both cases, potentialism is then introduced as a theory that makes these modal conceptions of sets precise.

As I see it, this approach can yield a strong argument for potentialism, provided that the intuitive conception of sets in question is itself true: potentialism is true because it is entailed by our intuitive conception of sets, which is itself true. This is the general form of what I call the 'argument from an intuitive conception' for

[33] Of note is that the potentialist who accepts classes as intensional entities can also accepts this (see footnote 3).

[34] These types of collections are discussed, e.g., in (Lévy, 1976).

[35] See, e.g., Warren (2017, 82f.), or references in (Incurvati, 2020, 71f.), and perhaps Linnebo (2010, 150). Berry (2022, 15) in the quote above also suggests that the modal version of the Axiom of Naïve Comprehension has strong intuitive appeal.

potentialism. To be successful, the argument from an intuitive conception thus has to be supplemented with an argument for why the relevant intuitive conception of sets is true.

An "intuitive conception of sets" is standardly understood to consist of basic or even axiomatic statements about sets couched in an at least partly informal language. For instance, the iterative conception of sets is usually taken to include the statements that sets are formed in stages, that at the first stage, there is an empty set, that whenever there are two sets the set that contains them is formed at the next stage, and so on. These basic statements about sets usually can't be justified inferentially or proved from other basic statements. This leaves broadly two options for arguing for their truth: via abductive arguments, on which an implicit conception of sets is true because it best serves certain theoretical and explanatory roles in mathematics or the broader sciences,[36] or via arguments from implicit definitions, on which an implicit conception of sets is true because it implicitly defines the primitive mathematical expressions in it (such as 'set' and 'membership').[37]

In my view, potentialists and actualists alike should opt for the latter option. Firstly, abductive arguments for the truth of an intuitive conception of sets often invoke the conception's ability to explain set-theoretic paradoxes;[38] but this would reduce the argument from an intuitive conception of sets to the argument from explanatory power, which, as I argued in §2, is unsuccessful (though, as I will explain in §3.4, explanatory considerations can still be relevant to the argument from an intuitive conception via an argument from implicit definitions). Secondly, potentialists and actualists alike need a metasemantic theory; that is, they need an account of what determines the meanings or extensions of their set-theoretic expressions. But it is widely accepted that something like an implicit definition theory is the only viable metasemantic theory for the language of mathematics.[39] Thirdly, as I will explain in §3.2, an argument from implicit definitions can help potentialists and actualists further motivate and develop the textbook explanations for the non-existence of contradictory sets that I have been invoking in §2 by revealing that they are analytic explanations. Finally, as I will explain in §3.1, an argument from implicit definitions can help potentialists explain their interpretations of their modality, and, in particular, it can help potentialists who want to use a primitive notion of modality defend and explain their interpretation. In sum, I think the potentialist's argument from an intuitive conception of sets is strongest and most fruitful when supplemented with an argument from implicit definitions for the truth of the intuitive conception in question. In the following §3.1, I will flesh out one such argument from implicit conception and explain its consequences for the debate between potentialism and actualism (in §§3.2–3.4). But note that even if one rejects the implicit definitions strategy in favor of the abductive strategy, my conclusion that the debate over actualism and potentialism can only be settled on the basis of pragmatic considerations will still stand. Indeed, if we set aside the virtue of explaining set-theoretic paradoxes (because of the arguments from §2), an abductive argument for one conception over another will invoke pragmatic virtues of simplicity, mathematical strength, usefulness, and so on, which are precisely the types of considerations I take to bear on the question of which meanings to assign to our set-theoretic expressions, as I will explain in §3.4.[40]

3.1. A descriptivist metasemantics

An *argument from implicit definitions* for the truth of some intuitive conception of sets says that this intuitive conception of sets is true because it is "intrinsically justified," or "analytically" true, or "implicitly defines" the primitive set-theoretic expressions (that is, the expressions that aren't explicitly defined). The details of the argument from implicit definitions won't make a difference to my overall claims about the debate between actualism and potentialism, but, for concreteness, let me outline a specific (and my preferred) way to run this argument. We start with a metasemantic view of how set-theoretic expressions get their meanings and extensions from their use, known variously as 'descriptivism', 'the method of implicit definitions', 'the Hilbertian Strategy', or 'the Ramsey–Carnap–Lewis method'.[41] On this metasemantic view, speakers use certain sentences in a

36 See, e.g., (Maddy, 1988, 2011), and more recently (Incurvati, 2020). The "anti-exceptionalist" views about logic and mathematics originating in (Williamson, 2013; Williamson and Armour-Garb, 2017) could be included in category.

37 See, e.g., recently (Warren, 2020) or (Soysal, 2020; Soysal, forthcoming).

38 E.g., Incurvati (2020, 64–69) does this.

39 See also my arguments in (Soysal, 2024a). Potentialists also generally accept this type of metasemantics; see, e.g., (Warren, 2017), (Linnebo, 2018, 33ff., 135ff.), or (Berry, 2022, 155ff.).

40 See again, e.g., (Incurvati, 2020, 44–69), (Maddy, 1988, 2011), (Williamson and Armour-Garb, 2017).

41 This is also a version of inferentialism, as explained in (Soysal, forthcoming) and (Warren, 2022).

privileged way, and their privileged use determines that these sentences are true (or, perhaps, that they are true if the use is consistent). The sentences that are used in this privileged way are thus in some sense "analytic" or "true in virtue of meaning." So, if the sentences that are part of some intuitive conception are used in the relevant privileged way, they are also thereby true in virtue of meaning.

Here is one way to spell out this kind of metasemantic view in a bit more detail.[42] On a descriptivist metasemantics, speakers associate primitive set-theoretic expressions with a certain theory or description, and these set-theoretic expressions thereby have meanings or extensions that make this associated theory true (if the theory is consistent).[43] The "speaker associations" here are intended to capture the speakers' use of the expressions: to associate some expressions $E_1, \ldots, E_n$ with a theory $T(E_1, \ldots, E_n)$ is, roughly, to be disposed to accept sentences in $T(E_1, \ldots, E_n)$ "come what may," that is, no matter what evidence one were to have.[44] "Accepting" some $\phi \in T(E_1, \ldots, E_n)$ can manifest itself in various ways; for instance, one can be disposed to utter ϕ, to say 'Sentence ϕ is true', to base one's reasoning on the truth of ϕ, and so on.[45] Two further clarifications will be relevant for the discussion that follows. Firstly, associated theories don't need to be in a formal, uninterpreted language; they can have meaningful expressions of ordinary language.[46] Expressions that occur in an associated theory but that aren't implicitly defined by that particular associated theory are what Lewis (1970) calls the 'old' terms of an associated theory or what I like to call 'anchors' (Soysal, 2020). So, in other words, a theory associated with our set-theoretic expressions can be "anchored" in ordinary language. (Moreover, the primitive set-theoretic expressions themselves don't need to be restricted to the formal symbol '$\in$'; they can be, say, 'set' and 'membership'.) Secondly, the descriptivist view leaves open that not all set theorists associate the same theory with set-theoretic expressions. Distinct associated theories T_1('set', 'membership') $\neq$ T_2('set', 'membership') could yield the same meanings and extensions for the primitive set-theoretic expressions in case these theories are "equivalent" in some strong sense (call this 'translational equivalence'). For instance, we would like to say in general for this type of metasemantic view that expressions that have mere typographical or grammatical differences can still have the same "privileged use," and, thus, the same meaning.[47] In such cases, we would thus like the respective associated theories to come out as translationally equivalent.[48] But, of course, distinct associated theories don't need to be translationally equivalent. Descriptivism thus also leaves open that different set theorists use the primitive set-theoretic expressions with different meanings (as well as, perhaps, different extensions).

With this descriptivist metasemantic view on the table, potentialists can say that among the statements that are part of the theory we associate with our set-theoretic expressions are modal statements, such as the statement that any things can form a set. These modal statements—and potentialism itself—are thus true because they are part of the theory we associate with our set-theoretic expressions. In other words, the potentialist can argue that their modal intuitive conception of sets is part of the theory associated with set-theoretic expressions, and, thus, that it is true (in virtue of meaning). This is the argument for potentialism from an intuitive conception of sets via the implicit definitions strategy. I think this type of argument is at least implicit in the remarks that potentialism uncovers something that is part of our intuitive conception of sets or that it captures our naïve intuitions about sets. And I think that this type of argument is very strong, because something like this metasemantic story must be correct for mathematical expressions.[49]

The key premise potentialists need to defend is that modal statements really are part of the theory we associate with our set-theoretic expressions. As I see it, potentialists have two options for making this case.

42 This is my preferred view, which I develop in (Soysal, 2020, 2021, 2024a, forthcoming; Kipper and Soysal, 2022).

43 For (Warren, 2020), for instance, consistency isn't even a requirement, Clarke-Doane (2020) discusses that Σ_1-soundness might be needed; see (Soysal, forthcoming) and (Picollo and Waxman, 2024) for discussion. It is standardly thought that the sentences in the theory conditionalized on the existence of entities that satisfy the theory are themselves automatically true (see, e.g., Lewis, 1970, and Soysal, forthcoming).

44 More precisely, it is to be disposed to accept them conditional on the existence of $E_1, \ldots, E_n$. For more discussion and explanation of the descriptivist view, see, e.g., (Soysal, 2020, forthcoming).

45 As explained in, e.g., (Soysal, forthcoming) or (Warren, 2020, 33ff.).

46 This is unlike on the "formalist" Hilbertian or Carnapian readings of the descriptivist metasemantics, which is very common; see, e.g., (Warren, 2020). As I argue in (Soysal, 2020, forthcoming), realizing that associated theories needn't be in a formal language where only the logical expressions are assumed to have meanings resolves numerous philosophical puzzles about mathematics.

47 See, e.g., the notion of "stylistic variance" discussed by (Warren, 2020, 139ff.), or notions of notational variance discussed by, e.g., (French, 2019; Button, 2021).

48 See (Soysal, 2024b) for the development of such a notion of translational equivalence.

49 As potentialists themselves agree, see footnote 39.

First, they can say that ordinary modal expressions such as 'can', 'possible', or 'necessary', occur in the theory we associate with our primitive set-theoretic expressions and they retain their standard meanings; in other words, the potentialist can say that the theory associated with set-theoretic expressions has modal anchors. The second option is to say that modal expressions are part of the primitive expressions implicitly defined by the theory associated with set-theoretic expressions; in other words, the potentialist can say that we associate a theory T('set', '∈', 'can') with our set-theoretic expressions together with an expression of set-theoretic or mathematical modality, 'can'. Here, the word 'can' thus gets a new set-theoretic or mathematical sense in virtue of this association. As I see it, this second option yields the best way to understand the potentialist's claim that her modality is primitive or idiosyncratic to set theory or mathematics, or the claim that the potentialist can only specify "structural features" that any interpretation of the modality should have (Linnebo and Shapiro, 2017, 171): for, on this option, the story of the potential hierarchy of sets itself determines the (structural features of) the sense of 'can' at issue. The first option, in turn, can capture the other interpretations of the modality: for instance, one can anchor one's associated theory in the constructive interpretation of 'can' on which a sentence such as "there can be more sets" in the associated theory is assumed to mean that there is a metaphysically possible world in which some social process has yielded further sets than there are in the actual world. (As I will argue in §3.4, the linguistic interpretation of 'can' is harder to make sense of on this picture and this counts against it.)

On either option, the potentialist needs to argue that certain modal statements about sets are part of the theory we associate with set-theoretic expressions, that is, that we hold certain modal statements about sets true come what may, that certain modal statements about sets are part of the basic story of the hierarchy of sets we accept non-inferentially and intuitively. But the problem is that some people—and, in particular, actualists—will reject that modal statements are part of the theory that *they* associate with their set-theoretic expressions. For actualists, any modal talk in the description of the iterative hierarchy of sets is metaphorical, and it is eliminated in the ultimate story that is held true come what may; after all, actualism as I understand it here is precisely the rejection of potentialism (§1). Whatever intuitive conception actualists associate with set-theoretic expressions, modal expressions are neither going to be anchors nor further primitives in their associated theory. On a more positive characterization, the theory actualists associate with sets could simply be any standard non-modal formulation of the iterative conception of sets.[50] It would likely include statements such as 'There is an empty set', 'Every set is well-founded', and so on—so, statements that can be formalized or formulated as axioms of set theory.[51] As I mentioned in §1, actualists can also further specify the universe of sets as either a proper class or a mere plurality. So, actualists's associated theory might include statements such as 'the universe of sets is a mere plurality'; it could thus have the notion of a plurality as either an anchor or a primitive in their associated theory. Then, the statement 'the plurality of all sets isn't a set' would be entailed by the theory that the actualist associates with her set-theoretic expressions. Alternatively, actualists could invoke the notion of a proper class—and perhaps even super-classes, and super-super classes, and so on—and hold true come what may statements such as 'The totality of all sets forms a proper-class', 'Proper classes aren't sets', 'Super-classes are neither sets nor proper classes', and so on. Either way, there is no modal language in the theory that the actualist associates with her set-theoretic expressions; from the current metasemantic perspective, we can even say that not having modal statements in the theory one associates with set-theoretic expressions is just what it is to be an actualist.

Actualists and potentialists thus associate different theories with their set-theoretic expressions. Unless their theories end up being translationally equivalent, this entails that actualists and potentialists mean different things by their set-theoretic expressions, and, thus, that the dispute between actualists and potentialists is merely

50 Some options include part of the conception outlined by Boolos (1971), or even a stage theory, e.g., articulated by Button (2021), or Incurvati's (2020, 61–64) "minimal" conception of sets.

51 Note that axioms themselves can be part of (or be entailed by) intuitive conceptions. In that case, an intuitive conception can "justify" an axiom if the intuitive conception is part of an associated theory: this would show that the axiom (or something that entails the axiom) is itself analytic. If axioms aren't themselves part of or entailed by some intuitive conception, then it is harder to account for how an intuitive conception can justify the truth of the axiom from the current perspective: if some intuitive conception C is part of an associated theory, and $C+A$ is a distinct, translationally non-equivalent theory, then strictly speaking the primitive set-theoretic expressions in C and in $C+A$ have distinct meanings; facts about C-sets won't justify facts about $C+A$-sets, and axiom A isn't true of C-sets. (This is the familiar point that Carnapian explications "change the subject.") As I will explain in §3.4, an alternative would be that the intuitive conception gives *pragmatic* motivation for an axiom A: one pragmatic reason to accept $C+A$ rather than $C+\neg A$ might be that the former is more similar to or captures the spirit of C. For further discussion, see, e.g., (Soysal, 2024a).

verbal. If the potentialist and actualist theories are translationally equivalent, then there is no disagreement between the actualist and potentialist, for they are merely choosing different words to express the same things; for instance, the potentialist uses ‘there can be’ for what the actualist expresses as ‘there is’. There is a current debate over whether the potentialist and actualist theories are translationally (or “notationally”) equivalent; for instance, Linnebo and Shapiro (2017, §7) argue that their theories aren’t mere notational variants of actualist ones if higher-order resources are involved, while Tim Button (2021) d discusses this and the “near-synonymy” of many versions of the potentialist theories and actualist theories, suggesting that near-synonymy might suffice for translational equivalence and thus for sameness of meaning. I won’t take a stand on this debate here.[52] I will only note that if potentialists are right, then this would entail, from the current perspective, that they mean something different with their set-theoretic expressions than the actualists, because the potentialist and actualist theories in question are associated, meaning-determining theories. Indeed, the argument from an intuitive conception for potentialism works precisely because the potentialist theory is (or is entailed by, or is a precisification of) an associated, meaning-determining theory. So, in either case, the dispute between the actualist and the potentialist isn’t as substantial as one might have thought. I will explain how the debate over potentialism and actualism can move forward from here in §3.4. But first, since we now have the actualist view supplemented with the implicit definitions strategy, let me clarify two things left open in our discussion in §2.

3.2. Textbook explanations as analytic explanations

First, we can illuminate the actualist’s textbook explanation for why there is no set of all non-self-membered sets (or of all cardinals, ordinals, and so on). The actualist can say that the fact that there is no such set follows from at least one principle that is part of the theory she associates with her set-theoretic expressions. For instance, surely, statements such as the Axiom of Foundation and the Axiom of Separation—or some other statements that entail ‘No set is a member of itself’—are going to be part of the theory actualists associate with their set-theoretic expressions. But this means that, whatever ‘set’ and ’membership’ mean, they mean something such that ‘No set is self-membered’ is (analytically) true. But then, it is a logical consequence of analytic truths that there is no set of all non-self-membered sets. This analytic explanation is as good an explanation as we can hope to get for why there is no set of all non-self-membered sets.[53]

To take an analogy, say (plausibly) that we associate the term ‘bachelor’ with some theory that includes ‘every bachelor is unmarried’, that is, we hold ‘every bachelor is unmarried’ to be true come what may. Then, if someone asks “But why isn’t there a happily married bachelor?” the explanation we should give is: “There isn’t a happily married bachelor because it is analytically true that every bachelor is unmarried, and it is a simply consequence of this analytic truth that there isn’t a happily unmarried bachelor.” This is as good as explanations ever get. And I contend that the actualist has just as good an explanation for why there is no set of all non-self-membered sets: there is no set of all non-self-membered sets because it is an analytic truth that no set is self-membered (given what we associate with our set-theoretic expressions), and it is a simple consequence of this that there is no set of all non-self-membered sets. This is as good as explanations get; it would be just as absurd to keep asking “Ok, but *why* is there no set of all sets?” as it would to keep asking “Ok, but *why* is there no happily married bachelor?”

The lingering worry articulated by Linnebo that we discussed in §2.1 should no longer have force. Consider the plurality of all sets. It follows from how we define ‘set’ that everything in that plurality is something that isn’t self-membered. That is, there are no self-membered sets in this plurality, and thus there is also no set of all the sets in this plurality. The same thing could be said about bachelors: Consider the plurality of all bachelors. There is no married person in this plurality, simply because of how we define ‘bachelor’.

It can be hard to dispel the idea that there is still something unexplained on the picture that I am proposing. Here is an attempt to capture this idea. Say we consider the plurality of all sets; but now we imagine that they are all “laid out” in front of us. On the face of it, one can then imagine that it is perfectly possible for there to

52 Although I argue that neither near-synonymy nor synonymy (or definitional equivalence) suffice for translational equivalence on a descriptivist metasemantics (Soysal, 2024b).

53 This cashes out the proposal I make in (Soysal, 2017a, 2017b) to understand conception-based explanations as analytic or conceptual explanations.

be one more set that contains all of these sets, laid out in front of us; we imagine "lassoing" all the sets laid out in front of us. But that is an illusion. If what is laid out in front of us really is the plurality of all sets, then all the sets are already out there in front of us. If one imagines, say, a (finite) list of sets, then one can genuinely imagine the set that contains all of the sets on this finite list. But that is very different from imagining *all sets* laid out in front of us. Think of another analogy. One can imagine the set of all natural numbers, "laid out" in front of us. On the face of it, one can then imagine that there is a natural number that is larger than any number in this set. But of course, there can't be. If what we are really imagining is the infinite series of the natural numbers, then there are no more natural numbers to be added to the series! On the actualist perspective, these are both instances of how our intuitions about infinities can be radically misleading.

Finally, note that potentialists have the same type of explanation available to them, using their own associated theory. In fact, the (non-modal) Axiom of Foundation itself is usually accepted as part of the potentialist conception of sets,[54] so the exact same explanation just discussed is also available to the potentialist. So, when it comes to explaining why there is no set of all non-self-membered sets—or no set of all ordinals, or all cardinals, and so on—actualists and potentialists can be exactly on a par.

3.3. Unexplained impossible existence on a primitive modality

The second point that we can now clarify is why actualists don't need to explain why there can't be a set of all sets on a sense of 'can' that is primitive to the potentialist. As I explained in §3.1, the best way to make sense of a primitive potentialist modality is to say that 'can' gets its meaning together with primitive set-theoretic expressions by being associated with a theory that involves them all. What this means is that the word 'can' has this primitive meaning only for speakers who are disposed to accept the potentialist theory come what may. Actualists aren't such speakers. This doesn't necessarily mean that the potentialist's sense of 'can' can't be translated into the actualist's language. But to avoid ambiguity, when the potentialist asks 'Why can't the plurality of all sets form a set?', the actualist should first read this as something like: 'Why shcan't the plurality of all shmets form a shmet?' One answer the actualist can then give to this question is that shmets are simply outside the scope of her theory. And if there is a translation of the potentialist's expressions into the actualist's, then the sentence 'The plurality of all shmactually existing shmets shcan form a shmet', which is (presumably) entailed by the potentialist's associated theory, should be translated into a sentence that is true given the actualist's associated theory, since an adequate translation should at least preserve analytic truths or truths entailed by the associated (meaning-giving) theory. But then, the actualist would also be able to explain why this fact holds about sets, given her associated theory. So, once again, the potentialist hasn't shown that there is an unexplained fact on the actualist picture.

3.4. A cost–benefit comparison of actualism and potentialism

Let us take stock. I argued that the best way to argue for potentialism is to say that it follows from an implicit definition of set-theoretic expressions. But then, given that there are other ways to implicitly define set-theoretic expressions—such as, in particular, the actualist's—the dispute between potentialism and actualism should be understood as merely verbal: Actualists and potentialists (plausibly) mean different things with their set-theoretic expressions, and, thus, they don't really disagree with each other over whether the universe of sets is potential. Rather, the universe of sets is potential on the potentialist's sense of 'set', while it isn't on the actualist's sense of 'set'. As with all verbal disputes, the natural next question is whether there is any reason to pick one meaning assignment over another. That is, is there reason to associate the potentialist theory with 'set' and 'membership' as opposed to the actualist theory? Or, if we disambiguate and associate, say, 'p-set' and 'p-membership' with the potentialist's associated theory and 'a-set' and 'a-membership' with the actualist's associated theory, is there reason to speak a language with 'p-set' and 'p-membership' as opposed to a language with 'a-set' and 'a-membership'? These kinds of questions can only be resolved by considering pragmatic (or perhaps normative) reasons. Indeed, there is no question of which choice gets at the truth, since both options will yield their respective (analytic) truths; that is, these aren't pragmatic reasons for or against the *truth* of

[54] See, e.g., (Linnebo, 2013, 216f.).

potentialism or actualism. For simplicity (and without loss of generality), let me only consider the question of which theory to associate with (and thus which meaning to assign to) the expressions ‘set’ and ‘membership’. I will conclude by considering some pragmatic reasons that bear on this question and suggest that these weigh against associating the potentialist theory with the primitive set-theoretic expressions; or, as I will say, “against being a potentialist.” As I explained in §§2.1–2.2, I take the most plausible versions of potentialism to be the ones with the primitive and the linguistic interpretations of the modality. I will thus only consider these two options in my cost–benefit comparison.

There are many different types of pragmatic reasons one can give for or against using certain mathematical expressions with certain meanings. I won’t aim to be exhaustive here, and my discussion can thus also be seen as an invitation to consider further pragmatic costs and benefits on behalf of potentialism and actualism. But here are what I take to be the broad categories of pragmatic criteria to consider: simplicity and ease of use, fruitfulness (mathematical, scientific, or perhaps even philosophical), faithfulness to some historical or pre-theoretical conception, and explanatory power (mathematical or otherwise). The arguments in §2 support that the potentialist language doesn’t have any explanatory advantage over the actualist language. As far as I know, no version of the potentialist theory promises to have useful consequences for the natural sciences, so I will set considerations of scientific fruitfulness aside. Most versions of the potentialist formal theories are at least mutually interpretable with standard ZFC (as I mentioned in §3.1, Button (2021) argues that many are even near-synonymous), and thus arguably don’t provide any new mathematical results (including ease of proof) over ZFC.[55] This suggests that actualism and potentialism are on a par in terms of mathematical fruitfulness (though simplicity and ease of expression can also help mathematically, and, as I will explain, those considerations favor actualism). The fact that non-modal ZFC is still the standard choice in the mathematical community might also suggest that potentialism isn’t more fruitful mathematically than actualism. In any case, the standard motivations for potentialism are philosophical, so more would need to be said to motivate the mathematical fruitfulness of potentialism. I thus also set considerations of mathematical fruitfulness aside here.

3.4.1. Pragmatic comparison of actualism vs. potentialism with a primitive modality

Consider, first, the option where the potentialist’s modality is primitive. I think one can make the case that it is a pragmatic cost to have one more primitive expression in one’s associated theory, especially if that expression already has a standard meaning in the background language. Having an extra such primitive adds some complication to the associated theory and ambiguity in the overall language, both of which constitute at least some pragmatic cost.

Take, next, the criteria of faithfulness to some historical or pre-theoretical notion. As I mentioned at the beginning of §3, some potentialists motivate their potentialist conception by saying that it captures the intuitive part of the naïve (and inconsistent) conception of sets, because the potentialist conception makes true the modal version of Naïve Comprehension: for instance, on the version concerning pluralities, this is the statement that every “plurality could form a set,” as Builes and Wilson (2022, 12f.) put it. Even granting that it is desirable to retain a version of Naïve Comprehension in a theory of sets,[56] I don’t see how a primitive notion of “can” could help capture any intuitive version of Naïve Comprehension. Arguably, ‘can’ in the potentialist’s associated theory is a modality in name only: For one, it isn’t any of the intuitive notions involved in any characterization of the intuitive idea of Naïve Comprehension. Moreover, as Andrew Bacon (2024) has argued in a slightly different context (concerning “width” potentialism), the potentialist’s primitive mathematical “modality” seems to flout some of the core orthodoxies about modality in general.[57] Potentialists would thus have to argue that there is nonetheless something of the intuitive idea that things “can” form sets that is retained in their statements such as ‘every plurality can form a set’, which seems to be a major challenge.

To summarize, there are some pragmatic costs to potentialism with a primitive modality, and there are no clear pragmatic benefits to it. The balance of pragmatic reasons seems to favor actualism.

[55] See, e.g., (Linnebo, 2013) and (Studd, 2013).

[56] I doubt this; indeed, it has also been argued that Cantor’s conception of sets is fundamentally different from the logicists’ “naïve” conception (see, e.g., Ferreirós, 2023), and so one might argue that our concept of set should be faithful to Cantor’s notion rather than to the naïve notion.

[57] Bacon (2024) shows that this “modality” radically flouts Brouwer’s principle in that it is possible that there are truths that are possibly impossible and Leibniz biconditionals (i.e., what is possible, in the broadest sense of possible, is what is true in some broadly possible world) (Bacon, 2024, 132).

3.4.2. Pragmatic comparison of actualism vs. potentialism with a linguistic modality

Second, consider the option of anchoring the potentialist's associated theory with a linguistic kind of modality. Some have argued that potentialists adopt some modal principles that conflict with the intuitive linguistic understanding of the modality at issue.[58] I think these concerns are serious, and, in our framework, they put pressure on whether the potentialist's modality really is anchored in a linguistic type of modality. A related worry is that, on a natural understanding of the potentialist's linguistic interpretation, the potentialist theory is merely describing how one could use the non-logical expressions of the language of set theory (that is, 'set' and '$\in$', or just '$\in$') with different (more and more expansive) meanings, without actually (at least explicitly) talking about sets.[59] On this understanding, the argument from an intuitive conception for potentialism as I have developed it wouldn't apply: the potentialist isn't implicitly defining a modal conception of sets, but, rather, she is theorizing about different kinds of possible semantic changes involving 'set' (that is, their theory is about 'set', and not *sets*).[60] On this interpretation, it is clear that actualists and potentialists are talking past each other: As I discussed in §§2.1–2.2, the actualist can grant that one could change the meaning of 'set', but she doesn't want to use 'set' with different meanings. Instead, she is theorizing about sets on her (fixed) use of 'set'. The actualist can capture the more expansive meanings that the potentialist is concerned with, simply with different words than 'set'—for instance, 'proper class', 'super class', and so on. So, what would be the pragmatic benefit of adopting a theory of semantic change as opposed to the actualist's theory of sets?

Berry (2024, 12) argues that the potentialist theories on this linguistic interpretation don't commit one to the existence of sets (since they only theories about how one could use language), and thus they could accommodate a nominalist metaphysics. Some might find this to be a philosophical advantage. But note that actualism, too, is consistent with various ontological views about the nature of sets (structuralism, thin realism, robust realism, and so on), depending on what else one decides to include in one's associated theory. Actualism could even be combined with the claim that sets don't exist, if it is understood, for instance, that an associated theory merely delineates analytic truths about sets that are conditional on the existence of sets (roughly such as "Carnap-sentences" in Lewis, 1970).[61]

A second reason to adopt the potentialist theory on this linguistic understanding might be that it adequately captures a real phenomenon of semantic change. But it is unclear whether anyone actually uses 'set' in this ever-changing way, and there seem to be reasons against speaking in this way. In general, there is some pragmatic cost to using a word with multiple different meanings, and to changing the meaning of a word midway through some piece of reasoning. At the very least, this complicates the semantic theory for that expression, and it can lead to miscommunication among speakers. But what the potentialist theory is capturing, on this view, is precisely the practice of changing the meanings of one's words, including right in the middle of a reasoning process. For instance, the idea is that in running through the reasoning of Russell's paradox, we shift the extension of our quantifiers and/or set-theoretic expressions (Warren, 2017, 102–110), which is why it is "possible" that there is a set of all those (actual) non-self-membered sets. If there is an alternative way of speaking without switching the meanings of our expressions, there would be some pragmatic reason to prefer it. But that is precisely what the actualist associated theory is giving us: we can use 'proper class' or 'mere plurality' for the collections that are sets on the expanded understanding of 'set', without changing the meaning of any of our expressions.[62]

Consider, finally, the criterion of faithfulness. Once again, I think it is unclear that potentialism here captures the intuition that there can be more sets. Surely, the intuition wasn't that one can have more sets

58 E.g., Berry (2022, 62f.) discusses how the "maximality" principle "[a]t every stage, all the entities that can be introduced are in fact introduced" that Linnebo adopts conflicts with the intuitive understanding of his linguistic modality, since we don't introduce all abstraction principles at once (see also Berry's similar points against Studd's linguistic modality (Berry, 2022, 63–65). Similarly, Warren (2017, 114f.) argues that the intuitive linguistic understanding makes little sense of iterated modalities or quantifying into modal contexts.

59 See, e.g., the views of (Linnebo, 2018) and (Studd, 2019).

60 Warren (2017) might be giving an alternative view on which the meanings of the set-theoretic expressions don't change but only the meanings of the quantifiers do. On this view, then, we could say that sets are implicitly defined in terms of different quantifiers, e.g., that Warren's NC+: $\exists^+ y \forall x (x \in y \leftrightarrow \phi(x))$ (or perhaps a version of it with a 'set' predicate) is part of the theory associated with set-theoretic expressions (Warren, 2017, 104).

61 See (Soysal, forthcoming) for discussion of the related problem of existence for descriptivism about the language of set theory. See also clarifications about this point in §3.1.

62 These are also reasons against adopting NC+ as part of our associated theory (see footnote 60).

because we can change the meaning of 'set' (or of the quantifiers). Rather, the intuition was that there can be more things *like those things*, the sets, so, more "collections" in some intuitive sense of 'collection' (and under the standard sense of the quantifiers). In contrast, it is available to the actualist to say that, although there aren't more sets on pain of contradiction, there are indeed more "collections": namely, proper classes, super-classes, and so on. Logic tells us that for no collection of type X, there is an X of all Xs. But for any collection of some type, there is some collection of some other type of all the collections of the former type. As I see it, this story captures the original intuition even better than the potentialist story on the linguistic interpretation (or on the interpretation with a primitive modality).

Once again, we saw some pragmatic costs to potentialism this time with a linguistic modality, and no clear pragmatic benefits. The balance of pragmatic reasons seems once again to favor actualism over potentialism.

4. Conclusion

This paper examined arguments for being a potentialist. The most common type of argument given for potentialism are arguments from explanatory power. I argued that these arguments are unsuccessful: either the potentialist is on a par with the actualist when it comes to explaining facts about sets, or the facts that are purportedly unexplained on the actualist picture should only be accepted if one is a potentialist to begin with. I then outlined what I take to be the most promising line of argument for potentialism: potentialism is true because it is part of the meaning-determining, analytic part of a theory of sets. The problem with this argument is that not everyone adopts (nor should adopt) this potentialist conception of sets. Moreover, as I argued, there are numerous pragmatic reasons against adopting the potentialist conception of sets over the default, actualist, conception of sets that underlies our standard mathematical practice. Why not, then, simply be an actualist?

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References

Bacon, A. (2023). Zermelian extensibility. Manuscript.
Bacon, A. (2024). Mathematical modality: An investigation of set theoretic contingency. *Journal of Philosophical Logic*, *53*, 131–179.
Barton, N. (2023). What makes a 'good' modal theory of sets. Unpublished manuscript.
Barton, N. (2024). Potentialist sets, intensions, and non-classicality. https://neilbarton.net/2024/06/29/project-potentialist-sets-intensions-and-non-classicality/. Accessed: 2024-06-30.
Benacerraf, P. & Putnam, H. (Eds.). (1983). *Philosophy of Mathematics* (2nd ed.). Cambridge University Press.
Berry, S. (2022). *A Logical Foundation for Potentialist Set Theory*. Cambridge University Press.
Berry, S. (2024). Potentialist set theory and the nominalist's dilemma. *Philosophical Quarterly*.
Boolos, G. (1971). The iterative conception of set. *Journal of Philosophy*, *68*, 215–231.
Boolos, G. (1984). To be is to be the value of a variable (or to be some values of some variables). *Journal of Philosophy*, *81*, 430–449.
Builes, D., & Wilson, J. M. (2022). In defense of countabilism. *Philosophical Studies*, *179*(7), 2199–2236.
Burgess, J. P. (2022). Review of Sharon Berry's A Logical Foundation for Potentialist Set Theory. *Notre Dame Philosophical Reviews*.
Burgess, J. P. (2004). E pluribus unum: Plural logic and set theory. *Philosophia Mathematica*, *12*(3), 193–221.
Button, T. (2021). Level theory, part 1: Axiomatizing the bare idea of a cumulative hierarchy of sets. *Bulletin of Symbolic Logic*, *27*(4), 436–460.
Cantor, G. (1899). Letter to Dedekind. In van Heijenoort, J. (Ed.), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Harvard University Press.
Clarke-Doane, J. (2020). *Morality and Mathematics*. Oxford University Press.
Drake, F. R. (1974). *Set Theory: An Introduction to Large Cardinals*. North-Holland Publishing Company.
Dummett, M. (1991). *Frege's Philosophy of Mathematics*. Harvard University Press.

Ferreirós, J. (2023). The early development of set theory. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Summer 2023 edition.
Fine, K. (2006). Relatively unrestricted quantification. In Rayo, A., & Uzquiano, G. (Eds.), *Absolute Generality* (pp. 20–44). Oxford University Press.
French, R. (2019). Notational variance and its variants. *Topoi*, *38*(2), 321–331.
Glanzberg, M. (2021). Unrestricted quantification and extraordinary context dependence? *Philosophical Studies*, *180*(5-6), 1–22.
Incurvati, L. (2012). How to be a minimalist about sets. *Philosophical Studies*, *159*(1), 69–87.
Incurvati, L. (2017). Maximality principles in set theory. *Philosophia Mathematica*, *25*(2), 159–193.
Incurvati, L. (2020). *Conceptions of Set and the Foundations of Mathematics*. Cambridge University Press.
Jech, T. (2003). *Set Theory: The Third Millennium Edition, Revised and Expanded*. Springer.
Kipper, J. & Soysal, Z. (2022). A Kripkean argument for descriptivism. *Noûs*, *56*(3), 654–669.
Koellner, P. (2011). Independence and large cardinals. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Summer 2011 edition.
Kunen, K. (1980). *Set Theory: An Introduction to Independence Proofs*. North-Holland.
Lévy, A. (1976). The role of classes in set theory. In G. H. Müller (Ed.), *Sets and Classes: On the Work by Paul Bernays* (pp. 173–216). North-Holland.
Lewis, D. (1970). How to define theoretical terms. *Journal of Philosophy*, *67*, 427–446.
Lewis, D. (1991). *Parts of Classes*. Blackwell.
Linnebo, Ø. (2010). Pluralities and sets. *Journal of Philosophy*, *107*(3), 144–164.
Linnebo, Ø. (2013). The potential hierarchy of sets. *Review of Symbolic Logic*, *6*(2), 205–228.
Linnebo, Ø. (2018). *Thin Objects: An Abstractionist Account*. Oxford University Press.
Linnebo, Ø. & Shapiro, S. (2017). Actual and potential infinity. *Noûs*, *53*(1), 160–191.
Linnebo, Ø & Shapiro, S. (2018). Modality in mathematics. In O. Bueno & S. A. Shalkowski (Eds.), *The Routledge Handbook of Modality*. Routledge.
Maddy, P. (1988). Believing the axioms. I. *The Journal of Symbolic Logic*, *53*(2), 481–511.
Maddy, P. (2011). *Defending the Axioms: On the Philosophical Foundations of Set Theory*. Oxford University Press.
Menzel, C. (2018). Modal set theory. In O. Bueno & S. A. Shalkowski (Eds.), *The Routledge Handbook of Modality*. London and New York: Routledge.
Parsons, C. (1977). What is the iterative conception of set? In R. E. Butts & J. Hintikka (Eds.), *Proceedings of the 5th International Congress of Logic, Methodology and Philosophy of Science 1975, Part I: Logic, Foundations of Mathematics, and Computability Theory* (pp. 335–367). Reidel. Reprinted in P. Benacerraf & H. Putnam (Eds.), *Philosophy of Mathematics* (2nd ed.). Cambridge University Press, 1983.
Picollo, L. & Waxman, D. (2024). Internalism and the determinacy of mathematics. *Noûs*, 1–20.
Roberts, S. Ultimate V. Unpublished manuscript.
Scambler, C. (2021). Can all things be counted? *Journal of Philosophical Logic*, *50*(5), 1079–1106.
Simmons, K. (2000). Sets, classes and extensions: A singularity approach to Russell's paradox. *Philosophical Studies*, *100*(2), 109–149.
Soysal, Z. (2017a). *Formal Analyticity* (PhD thesis). Harvard University.
Soysal, Z. (2017b). Why is the universe of sets not a set? *Synthese*, *197*(2), 575–597.
Soysal, Z. (2020). Descriptivism about the reference of set-theoretic expressions: Revisiting Putnam's model-theoretic arguments. *The Monist*, *103*(4), 442–454.
Soysal, Z. (2021). From metasemantics to analyticity. *Philosophy and Phenomenological Research*, *103*(1), 57–76.
Soysal, Z. (2024a). A Carnapian solution to the puzzle of extrinsic justifications. In S. Arbeiter & J. Kennedy (Eds.), *The Philosophy of Penelope Maddy* (Vol. 31 of Outstanding Contributions to Logic). Springer.
Soysal, Z. (2024b). The metalinguistic construal of mathematical propositions. Manuscript.
Soysal, Z. (forthcoming). The problem of existence for descriptivism about the reference of set-theoretic expressions. In C. Antos-Kuby, N. Barton & G. Venturi (Eds.), *The Palgrave Companion to the Philosophy of Set Theory*. Palgrave.
Studd, J. (2013). The iterative conception of set: A (bi-)modal axiomatisation. *Journal of Philosophical Logic*, *42*(5), 695–725.
Studd, J. (2019). *Everything, More or Less: A Defence of Generality Relativism*. Oxford University Press.
Uzquiano, G. (2003). Plural quantification and classes. *Philosophia Mathematica*, *11*(1), 67–81.
Uzquiano, G. (2015). Varieties of indefinite extensibility. *Notre Dame Journal of Formal Logic*, *56*(1), 147–166.
Warren, J. (2017). Quantifier variance and indefinite extensibility. *Philosophical Review*, *126*(1), 81–122.
Warren, J. (2020). *Shadows of Syntax: Revitalizing Logical and Mathematical Conventionalism*. Oxford University Press.
Warren, J. (2022). Inferentialism, conventionalism, and a posteriori necessity. *Journal of Philosophy*, *119*(10), 517–541.
Williamson, T. (1998). Indefinite extensibility. *Grazer Philosophische Studien*, *55*(1), 1–24.
Williamson, T. (2013). *Modal Logic as Metaphysics*. Oxford University Press.

Williamson, T. & Armour-Garb, B. (2017). Semantic paradoxes and abductive methodology. In B. Armour-Garb & F. Kroon (Eds.), *Reflections on the Liar*. Oxford University Press.

Yablo, S. (2006). Circularity and paradox. In T. Bolander, V. F. Hendricks & S. A. Pedersen (Eds.), *Self-Reference* (pp. 139–157). CSLI Publications.

Zermelo, E. (1930). Über Grenzzahlen und Mengenbereiche. *Fundamenta Mathematica*, *16*, 29–47.