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Carnapian Logicism and Semantic Analyticity

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Abstract: This article argues for a (quasi-)Carnapian version of logicism about mathematics: there is a logicist conceptual framework in which (i) all standard mathematical terms are defined by logical terms, and (ii) all standard mathematical theorems are (likely to be) analytic. Along the way, the article explains the historical-philosophical background, how the definitions in (i) are to proceed, what the framework and the semantic notion of analyticity-in-a-framework are like, and why the probabilistic qualification 'likely to be' is used in (ii). The upshot is not some logicist epistemic foundationalism about mathematics but the insight that mathematics can be rationally reconstructed as being conceptual, i.e., as coming along with a conceptual framework.

Keywords: Logicism, mathematics, Carnap, analytic, probability.

1. Introduction

In the famous Königsberg conference from 1930, in which Arend Heyting presented intuitionism and John von Neumann formalism, Rudolf Carnap gave a lecture on logicism about mathematics which appeared as Carnap (1931) later. In his lecture, Carnap stated the logicist thesis in the following, still fairly standard, two-part manner:[1]

> 1. The *concepts* of mathematics can be derived from logical concepts through explicit definitions.
> 2. The *theorems* of mathematics can be derived from logical axioms through purely logical deduction. (Carnap, 1931, pp. 91f)

In line with Carnap, I am going to understand by *traditional logicism* about a mathematical language $\mathcal{L}$ and a mathematical theory $T_{\mathcal{L}}$ (formulated in $\mathcal{L}$) the conjunction of the following two theses:

[1] Some more recent (neo-)logicists, such as Hale and Wright (2001), might expand 'logical concepts' to 'logical or abstraction concepts' in 1, and some logicists might replace 2 by 'The *truths* of mathematics are logical truths (or analytic)'. For a survey of logicism, see Tennant (2013).

1a. All mathematical terms in $\mathcal{L}$ are explicitly definable from logical terms.
2a. All mathematical theorems in $T_{\mathcal{L}}$ are logically derivable from logical axioms and explicit definitions (the definitions claimed to exist by 1a).

Both Frege's (1884; 1893/18931903) logicism about arithmetic and Whitehead and Russell's (1910-1910 1913) logicism about general mathematics may be understood as aiming at traditional logicism in that sense, with suitable choices of $\mathcal{L}$ and $T_{\mathcal{L}}$.

Moreover, if one follows Frege (e.g. §3 of his *Grundlagen der Arithmetik*, 1884) in defining a sentence to be analytic just in case it is logically derivable from logical axioms and explicit definitions, one may reformulate 2a above in the equivalent manner:

2b. All mathematical theorems in $T_{\mathcal{L}}$ are (Frege-)analytic.[2]

Indeed, for any logicist whatsoever it is perfectly clear that definitions are just as indispensable for their own project as they are for mathematical practice itself. Since, according to our present-day understanding of logic, definitions are neither logical axioms nor logical rules, the logicist goal is thus not to show that mathematics is purely logical but rather that mathematics is analytic. When Carnap's thesis 2 above speaks of mathematical theorems being derivable from logical axioms through purely logical deduction, he simply understands 'logical' broadly enough to encompass also definitions.

Fast-forwarding more than thirty years to Carnap's autobiography in his Schilpp volume, Carnap describes his early encounter with Frege's logicism again in very similar terms:

> I had learned from Frege that all mathematical concepts can be defined on the basis of the concepts of logic and that the theorems of mathematics can be deduced from the principles of logic. Thus the truths of mathematics are analytic in the general sense of truth based on logic alone. (Carnap, 1963, p.46)

As the 'I had learned' suggests, Carnap retained his logicist convictions until that final stage of his career. But of course it is important not to overlook the 'can' here: for he had argued in his *Logical Syntax of Language* (Carnap, 1934, 1937) that mathematics could also be understood differently if reconstructed in an alternative framework (e.g. mathematical terms might instead be regarded as primitive non-logical terms; see Carnap, 1934, 1937, §84 and also §78). Carnap recommended to be tolerant about frameworks, as for him there was no fact of the matter which of them would be the "right" one to reconstruct mathematics within: there was a plurality of suitable formally precise frameworks available, and whether and how one preferred to reconstruct mathematics in any one of them reduced to the practical question of what choices would serve the specific aims of one's reconstruction to greater extent. However, it should also be clear that Carnap regarded the logicist reconstruction of mathematics in a logicist framework to be one of the options available, and he took it to be his preferred option for many salient purposes.[3]

[2] For a different understanding of Frege-analyticity, see Boghossian (1996).
[3] For more on Carnap's logicism, see Bohnert (1975) and, more recently, Marschall (2021). See Schiemer (2022) for a survey of logicism in logical empiricism more broadly.

In what follows, I will take up some of Carnap's ideas about theoretical terms, conceptual frameworks[4], and analyticity in order to develop a distinctively (quasi-) Carnapian version of logicism about mathematics. The ambition is not historical, but rather the goal is the systematic development and defense of a version of logicism on broadly Carnapian grounds.

The corresponding logicist thesis I want to argue for is:

There is a logicist conceptual framework, such that

1c. all standard mathematical terms are explicitly defined from logical terms in the framework,

2c. all standard mathematical theorems are likely to be (Carnap-)analytic, that is, semantically analytic, in the framework.

The term 'standard' in 1c and 2c is meant to apply to (reconstructions of) almost all terms and theorems of present-day pure mathematics. This will come about by rationally reconstructing, on logicist grounds, the language of second-order ZF set theory, which is known to allow for the definition of all standard mathematical terms used by pure mathematicians, and also the axiomatic system of second-order ZF set theory, which is known to allow for the derivation of all standard theorems proven by pure mathematicians so far. I will presuppose the deductive system of second-order logic (with Choice) to be genuinely logical and the second-order universal quantifier to range over all classes and class-relations of first-order individuals. The outcome will be a rational reconstruction[5] of pure set theory and indeed pure mathematics in the sense of clarifying, precisifying, systematizing, and interpreting set theory and mathematics from a logicist point of view while remaining close to set-theoretic and mathematical practice (though slight deviations from that practice are allowed for the sake of other virtues). The discussion of the logicist understanding of applied mathematics and of the role of mathematics in the empirical sciences and engineering will have to be left for a different occasion.[6]

Set theory is widely accepted by mathematicians to be one possible foundation for almost all of today's mathematics. That is: it is widely held that all standard mathematical terms are explicitly definable using only logical terms and the membership predicate $\in$, and that all standard mathematical theorems are derivable from first-order or second-order ZF with Choice and hence are true in all standard models of ZF with Choice or indeed true in *the* intended model (if there is just one). Call this common view 'ZFCism'.[7] What will Carnapian logicism add to this? ZFCism is a purely mathematical view that is not by itself a philosophical interpretation of mathematics and is compatible with different such interpretations. E.g., it might be given a special kind of realist interpretation of the following sort: metaphysically, sets and membership might be assumed to exist independently of reasoners and language, and set-theoretic truths might be assumed to be metaphysically necessary. Epistemologically, sets and membership

[4] For much of his career, Carnap would have spoken of constitution systems, languages or linguistic frameworks, though sometimes he also used the term 'conceptual framework', as in "many problems concerning conceptual frameworks seem to me to belong to the most important problems in philosophy" (Carnap, 1963, p.862). I prefer the term 'conceptual framework' in order to make clear that frameworks in that sense are not just syntactical but also involve semantic rules and semantic interpretation mappings. Conceptual frameworks correspond to the semantical systems or intensionally interpreted languages that became central to Carnap's work once he had taken his semantic turn, as exemplified by Carnap (1942, 1947/1956). In their most general form, conceptual frameworks encode syntactic, logical, semantic, pragmatic, epistemic, ontic, and other choices, and their construction, study, and application is philosophically useful (contra Maddy, 2007, Chapter 5) whenever philosophical concepts, theses or arguments depend on such choices. The logicism of this paper does depend on such choices.

[5] See Leitgeb and Carus (2024, Supp. D) for more on Carnap on rational reconstruction.

[6] But Carnap (1934, 1937, §84) rightly stresses that securing the applicability of mathematics to the empirical world is itself a vital part of the logicist project.

[7] I owe this terminology to an anonymous reviewer.

might be assumed to be epistemically accessible by quasi-perceptual means through which set-theoretic statements can be justified. Semantically, the membership predicate might be taken to be primitive,[8] to have its intended interpretation(s) in virtue of certain non-semantic facts, and the set-theoretic axioms might be regarded as synthetic. And so forth. Carnapian logicism will differ substantially from any such realist interpretation: it will stay closer to ZFCism itself, adding only a definition of sethood and membership in logical terms, assuming their interpretations to satisfy the set-theoretic axioms and to otherwise remain arbitrary, and regarding the set-theoretic axioms to be (likely to be) analytic. The resulting interpretation of set theory will be "thin" or "deflationary" in a sense similar to deflationary theories of truth in which the truth predicate is regarded as a (quasi-)logical expression the interpretation of which is only assumed to satisfy the Tarskian truth scheme for the object language in question. No substantial metaphysical or epistemological assumptions are assumed by any such deflationary conception of truth, and no such assumptions will be assumed by Carnapian logicism either. And the purpose of developing such a logicist interpretation in precise terms by rationally reconstructing mathematics in a logicist framework are strictly philosophical, not mathematical: the goal is not to give mathematicians a mathematically better foundation to work with—just as the realist view sketched before would not give mathematicians a mathematically better foundation—but rather to show that mathematics can be understood to be purely conceptual.

More generally, *(quasi-)Carnapian logicism* about a mathematical language $\mathcal{L}$ and a mathematical theory $T_{\mathcal{L}}$ is given by:

There is a logicist conceptual framework, such that

1d($\mathcal{L}$). all mathematical terms in $\mathcal{L}$ are explicitly defined from logical terms in the framework,

2d($T_{\mathcal{L}}$). all mathematical theorems in $T_{\mathcal{L}}$ are likely to be (Carnap-)analytic, that is, semantically analytic, in the framework.

Hence, 1c and 2c from before will follow from instantiating the schemes 1d and 2d with the names of the language $\mathcal{L}^2_{\in,Set}$ and the axiomatic system $ZF2[\in, Set]$ of second-order ZF. Focusing on these particular instances 1d($\mathcal{L}^2_{\in,Set}$) and 2d($ZF2[\in, Set]$) will prove useful for my logicist purposes; in particular, it will be convenient in so far as second-order set theory—just as its first-order variant—is regarded as at least one possible foundation for modern mathematics anyway. But most of the logicist project of the present paper could be carried out just as well for many other choices of $\mathcal{L}$ and $T_{\mathcal{L}}$. Therefore, readers are very much invited to apply the general logicist strategy of this article to other such choices, whether they concern alternative foundations of mathematics or, in a more piecemeal fashion, languages and theories for specific areas of mathematics (logicism about second-order Dedekind-Peano arithmetic, logicism about second-order Dedekind real analysis,. . .).

The definitions backing up 1d($\mathcal{L}^2_{\in,Set}$) will rely on an understanding of set-theoretic membership and sethood as theoretical concepts—concepts given by the axiomatic theory of second-order ZF—and on the corresponding explicit definitions of $\in$ and Set by purely logical higher-order epsilon terms (Section 2). The logicist framework in question, which is to be distinguished from proper scientific theories that can be formulated within the framework, will involve an object-linguistic and a metalinguistic part, both of which will

[8] In the usual axiomatic systems for set theory, '$\in$' is of course indeed a primitive. But this might change in a philosophical interpretation of set theory.

be based on higher-order logic and the logic of epsilon terms. The metalinguistic part will include semantic rules for the object-linguistic part, and a framework-relative semantic concept of analyticity will be introduced that is going to reflect these semantic rules (Section 3). Analyticity in that semantic sense will be entailed by Frege-analyticity, that is: logical axioms and explicit definitions in the object language of the framework and what is logically derivable from them in the framework will follow to be semantically analytic in the framework. But Carnapian semantic analyticity in the framework extends beyond Frege-analyticity in the framework. In other words: Frege-analyticity is sound but not complete with respect to semantic analyticity.[9] As we are going to see, the analyticity of second-order ZF in the semantic sense will depend on whether a certain second-order existence statement in the metalinguistic part of the framework holds true, which we will find very likely to be the case (Section 4). Accordingly, in contrast with traditional logicism, 2d($ZF2[\in, Set]$) does not claim the analyticity of standard mathematical theorems to be *derivable* from uncontroversial principles but just that these theorems are *likely to be* semantically analytic in the framework. Replacing the derivability of analyticity by its high probability should not come as too much of a surprise, as the analyticity of second-order ZF would entail its consistency, and we know from the Incompleteness Theorems that the consistency of second-order ZF could not be derived on more elementary grounds (assuming second-order ZF is consistent). Finally, I am going to draw some conclusions on what this (quasi-) Carnapian version of logicism does or does not show philosophically (Section 5). In particular, it would not serve any logicist version of epistemological foundationalism that would ask for logic to deliver a more secure foundation for mathematics. Instead, the main conclusions will be: mathematics can be rationally reconstructed as purely conceptual, that is, as coming along with a conceptual framework. In one version of Carnapian logicism, this includes the existence of abstract logical objects that are introduced by the logicist framework itself. And, at the very least, this reconstruction does not fare worse than any other philosophical interpretation of mathematics available, as it is formally clear, precise, and systematic, it remains close to mathematical practice, and it is philosophically coherent.[10]

Last but not least, I should stress that I have been qualifying my approach as *quasi*-Carnapian. The reason for this is that Carnap himself did *not* develop or defend logicism in this manner. Instead, for most of his logical and philosophical work, he relied on a version of the simple

[9] That is one reason why Carnapian logicism differs, e.g., from the conventionalism put forward by Warren (2020) who regards conventions as *syntactic* rules of language use. Another reason is that Warren's project is not one of rational reconstruction.

[10] The contemporary literature that comes closest to the theory to be presented are, first, Woods' (2014, Section 4.3) and Boccuni and Woods' (2020) version of abstractionist neo-logicism, second, Leitgeb, Nodelman, and Zalta's (2025) object-theoretic logicism, and third, Soysal's (2025) meta-semantic descriptivism. Woods and Boccuni advocate a neo-logicism in which abstraction operators in neo-logicist abstraction principles are given by second-order epsilon terms or, in any case, are semantically "arbitrary". The main differences to the present theory are: their philosophical background and interests consist in a combination of mathematical structuralism with Hale and Wright's (2001) neo-logicism based on abstraction principles (such as Hume's Principle); and they neither use a Carnapian concept of semantic analyticity nor argue for their basic principles to be analytic. Leitgeb, Nodelman, and Zalta (2025) also regard mathematical concepts as theoretical concepts given by mathematical theories. The differences are: they presuppose higher-order object theory as their background logic, which involves two kinds of predication; they do not invoke epsilon terms; and they do not apply a Carnapian concept of semantic analyticity. Instead, they combine Frege-analyticity with an extended notion of logical truth according to which a formula of a formal language is logically true just in case it is true in all models that include everything required for the possibility of having logically complex thoughts expressible in that language. As I will argue in Section 3, Carnapian frameworks are also meant to supply what is required for thought, and semantic analyticity in a framework tracks what the framework supplies. Finally, Soysal (2025) develops a version of meta-semantic descriptivism about logical and mathematical expressions in which these expressions have their meaning at least partially in virtue of descriptions; in the case of the membership predicate, the description is given by set-theoretic axioms. The differences from the present approach are: Soysal's theory is not a rational reconstruction but deals with the actual metasemantics of logical and mathematical language; it does not involve epsilon terms; and it is not meant to be based solely on Carnapian grounds (though overlapping with Carnap on theoretical terms and analyticity).

theory of types as his preferred logical system, which he described in formal detail in his *Abriß der Logistik* (Carnap, 1929), and from which parts of modern mathematics can be derived at least conditionally (that is, given certain assumptions, such as an Axiom of Infinity—see Carnap, 1929, Section 24e; I will return to this in Section 2). In fact, Marschall (2024, Section 3.2) presents historical reasons to believe that Carnap regarded our pre-theoretic understanding of set-theoretic membership to be sufficiently clear and determinate—much like Frege might have thought about the concept of extension—so that there would be no need to regard it as being determined by an axiomatic theory. But then again, as Bohnert (1975, p. 210) cites his conversation with Carnap in 1968, "He [Carnap] still thought set theory could be given an analytic interpretation".

In any case: what Carnap himself would have thought about the project of this paper is orthogonal to its strictly systematic ambitions. For me, the more interesting point is that Carnap *could* have thought of mathematics in the logicist manner I am going to describe, since he did have the philosophical resources to so so. And even more importantly: everyone else is invited to think of mathematics in the same manner, and if one did, one would be able to do so coherently.

2. Defining Membership and Sethood Logically

As mentioned in the introduction, the starting point of our considerations is the axiomatic theory

$$ZF2[\in, Set]$$

that is, classical second-order Zermelo-Fraenkel set theory (see Shapiro, 1991, p. 85)[11], which is formulated in the language $\mathcal{L}^2_{\in,Set}$, that is, with the logical and auxiliary symbols of classical second-order logic, the primitive descriptive binary membership predicate $\in$, and the primitive descriptive unary predicate Set for sets. Without further argument and Quinean worries notwithstanding, I will take the operators of pure second-order logic to be properly logical symbols, the axioms of the deductive system of second-order logic to be properly logical truths, and the rules of the deductive system of second-order logic to be properly logical valid.[12] The role of Set is just to restrict all first-order and second-order quantifiers in the axioms to sets, and to restrict the relata of the membership relation to sets.[13] In addition, I am also going to assume the Axiom of Extensionality for second-order entities to belong to the system of second-order logic; consequently, e.g., second-order property variables may be thought of as ranging over extensional properties or classes. And I will regard a second-order version of the Axiom of Choice to be included in the deductive system of second-order logic (following Shapiro, 1991, p. 67), without defending its logicality here.

[11] '$ZF2[\in, Set]$' can be used to denote the set of axioms of second-order set theory or the deductively closed set of formulas that are derivable from these axioms in the deductive system of second-order logic. The context should always make clear which of the two is meant in each case. In any case, I do not mean the set of formulas that are second-order consequences of these axioms in the model-theoretic sense. Similarly, I will leave it to the context to determine whether '$\in$' and 'Set' denote predicates, that is, linguistic items, or the concepts expressed by these predicates, or the extensions of these concepts.

[12] The deductive system of second-order logic may be viewed as a many-sorted variant of first-order logic and should thus be compatible even with a Quinean conception of logic.

[13] So all first-order quantifier occurrences in $ZF2[\in, Set]$ are of the form $\forall x(Set(x) \to \ldots)$ or $\exists x(Set(x) \land \ldots)$, the second-order Axiom of Replacement begins with $\forall f(\forall x(Set(x) \to Set(f(x))) \to \ldots)$, and the statement $\forall x, y(x \in y \to Set(x) \land Set(y))$ is accepted as yet another axiom. Note that overall $\forall x(Set(x) \leftrightarrow \exists y\, x \in y)$ becomes derivable. Given our aim of reconstructing pure mathematics, there will be no need to consider sets of urelements, that is, sets of non-sets. In a context in which no other entities were relevant than sets, the Set predicate could of course be eliminated, as is the case in Shapiro (1991, p. 85).

One of the advantages of going second-order is that second-order quantification makes the usual axiom schemes of first-order set theory obsolete, so that the axioms of $ZF2[\in, Set]$ may be regarded to form one longish but finite conjunction. But $ZF2[\in, Set]$ exhibits also other attractive features: almost all proven theorems of pure mathematics are known to be derivable from the axioms of $ZF2[\in, Set]$ and auxiliary definitions in the deductive system of second-order logic with Comprehension and Choice. Indeed, $ZF2[\in, Set]$ is a non-conservative extension of first-order ZFC, which, in turn, is often regarded as a foundation of modern pure mathematics. However, historically, Zermelo (1930) had formulated set theory (with urelements) in second-order terms, and second-order set theory seems to be closer to mathematical practice than its first-order version (see Shapiro, 1991, Sections 5.3–5.4). Like ZFC, $ZF2[\in, Set]$ captures the cumulative hierarchy of sets by proving that every set occurs in a hierarchy that is indexed by ordinals and given by $V_0 = \varnothing$, $V_{\alpha+1} = \wp(V_\alpha)$, and $V_\lambda = \cup_{\alpha<\lambda} V_\alpha$. In addition, unlike ZFC, $ZF2[\in, Set]$ is "almost" categorical, that is, it pins down the structure of the cumulative hierarchy uniquely up to its strongly inaccessible ordinal height (see Shapiro, 1991, p.86, and, for internal categoricity, Väänänen and Wang, 2015). This holds even though $ZF2[\in, Set]$ is of course deductively incomplete, as follows from the Incompleteness Theorems (assuming that $ZF2[\in, Set]$ is consistent). Moreover, for a logicist endeavor, it is reassuring that the basic individual entities described by $ZF2[\in, Set]$ are governed by the clear and precise identity criterion of first-order extensionality, they are similar in that way to Frege's extensions (which Frege regarded as logical objects), and they might even be viewed as intensionally rigid logical properties, so that e.g. a set $\{\varnothing, \ldots\}$ could be identified with the logical property $\lambda x(x = \varnothing \vee \ldots)$, the set $\varnothing$ with the logical property $\lambda x(x \neq x)$, and the like.[14]

Independently of whatever else the predicates $\in$ and Set might have meant antecedently, let us from now on think of $ZF2[\in, Set]$ as "implicitly defining" $\in$ and Set jointly with their underlying iterative conception of sets, where the iterative conception is preferably understood in a minimalist or deflationary manner (see Incurvati, 2020, Chapter 2 and especially Section 2.6). The fact that $ZF2[\in, Set]$ captures the cumulative hierarchy and is quasi-categorical goes some way towards making this plausible. Then what the theory $ZF2[\in, Set]$ does, next to its explicit or implicit existential claims about sets, is to determine the meanings of $\in$ and Set from their conceptual roles vis-a-vis the remaining meaningful symbols in $ZF2[\in, Set]$, that is, the logical symbols. And for these logical symbols I will take for granted that their meanings are fixed and determined uniquely. In particular, $\forall$ indeed means *for all*, whether on the first-order level of all individuals or on the second-order level of all classes, relations, and functions.[15] On that basis, the first step of our logicist reconstruction will consist in making the "implicit definition" of $\in$ and Set by $ZF2[\in, Set]$ fully explicit. In the remainder of this section, I am going to explain the idea of how this can be done, while the concrete implementation of that idea in a logicist framework will be carried out in the next section.

Now, what does it mean to understand $\in$ and Set so that all there is to them is given by $ZF2[\in, Set]$? Consider the open formula

$$ZF2[R, S]$$

[14] Carnap (1956, §23) discusses this option of reducing extensions to what he calls "L-determinate intensions". The option would not assume that all of these intensionally determinate logical properties could be expressed linguistically, of course.

[15] This is compatible with the deductive system of second-order logic being incomplete with respect to the "full" standard (model-theoretically defined) semantics for second-order logic. The deductive incompleteness of the system does not entail the expressive incompleteness of its language.

that results from replacing all occurrences of $\in$ in $ZF2[\in, Set]$ by the binary relation variable R and all occurrences of Set in $ZF2[\in, Set]$ by the unary class variable S. $ZF2[R, S]$ thereby expresses a constraint on the values of R and S. The idea will be to use $ZF2[R, S]$ to give a rigorous answer to the previous question by defining $\in$ and Set to be *an R and an S, respectively, such that* $ZF2[R, S]$. Other than satisfying the constraint expressed by $ZF2[R, S]$, the meanings of $\in$ and Set will be left arbitrary.

In more formal terms: let us assume our logical vocabulary to include Hilbert's indefinite description operator ϵ (see Hilbert and Bernays, 1934/19341939), both on the first-order and on the second-order level. Just as the standard definite description operator ι can be used to denote something by describing *the* entity that has such-and-such a property, ϵ can be used to denote something by describing *an* entity that has such-and-such a property. Thus, the epsilon operator is just like the iota operator but with the uniqueness presupposition stripped away. When there is more than one entity with the relevant property, the epsilon operator is instead understood to "pick" *any* of these entities. Accordingly, for first-order epsilon terms $\epsilon x \varphi[x]$ ("an x, such that $\varphi[x]$") and second-order epsilon terms $\epsilon R \psi[R]$ ("an R, such that $\psi[R]$"), the following two schemes comprise the logic of the epsilon operator (the so-called epsilon calculus, see Avigad and Zach, 2024):

Logical Axiom Scheme for First-Order Epsilon Terms:
$\vdash \exists x \varphi[x] \rightarrow \varphi[\epsilon x \varphi[x]]$.
Logical Axiom Scheme for Second-Order Epsilon Terms:
$\vdash \exists R \psi[R] \rightarrow \psi[\epsilon R \psi[R]]$.

When the antecedent of an instance of either of the schemes is false, so that there is no entity with the required property, no constraint is being imposed on what gets "picked" by the respective epsilon term.[16] With the exception of cases in which $\varphi[x]$ or $\psi[R]$ describes its respective x or R uniquely, I neither assume that there is a fact of matter of what is being "picked" by the respective epsilon term nor that there is a metasemantic mechanism that would determine what is being "picked". The idea is rather to view, e.g., the choice expressed by $\epsilon R \psi[R]$ as being describable in metalinguistic natural language terms by '$\epsilon R \psi[R]$ chooses *some/any/whatever* R, such that $\psi[R]$', which does not ascribe any fixed or determinate denotation to $\epsilon R \psi[R]$.[17]

While the Hilbert school used first-order epsilon terms in their efforts to carry out Hilbert's formalist program, Carnap (1959, [2000]) proposed to invoke second-order epsilon terms for the rational reconstruction of theoretical terms in science (see also Carnap, 1961). But before explaining Carnap's proposal in more detail, let me first state how second-order epsilon terms can be used to define $\in$ and Set explicitly.

I will present these definitions in two versions, the first of which is:

Definition 1. (Definition of $\in$ and Set, First Version)

(i) $\in =_{df} \epsilon R \exists S\, ZF2[R, S]$.
(ii) $\forall x$: $Set(x) \leftrightarrow_{df} \exists y\, x \in y$.

[16] One might also assume an extensionality axiom for ϵ to belong to the logic of ϵ, but it will not be relevant in what follows.

[17] If the metalanguage gets formalized itself, this amounts to interpreting object-linguistic epsilon terms by means of metalinguistic epsilon terms, which is much like the common practice of, e.g., stating the truth conditions of object-linguistic negation sentences with the help of the metalinguistic negation sign; see Section 3. See Leitgeb (2023) for a general semantic and metasemantic treatment of languages with semantically indeterminate expressions by means of metalinguistic epsilon terms.

Here, $\in$ is defined to be an(y) R, such that there is an S, such that $ZF2[R, S]$. And it is easy to see that, if there is such an R, the class Set is effectively defined to be the field of the relation $\in$ that has been defined by 1.[18] Thus, $\in$ and Set are defined by describing their conceptual roles in $ZF2[\in, Set]$, as promised.

The second version invokes a predicate '$Logical$-in-$\mathfrak{C}$' that is regarded as a primitive logical term by which logical individuals can be distinguished from non-logical ones in the conceptual framework $\mathfrak{C}$, where '$\mathfrak{C}$' denotes the forthcoming logicist framework:

Definition 2. (Definition of $\in$ and Set, Second Version)

(i) $\in =_{df} \epsilon R\, \exists S(\forall x(S(x) \to Logical\text{-}in\text{-}\mathfrak{C}(x)) \land ZF2[R, S])$.
(ii) $\forall x$: $Set(x) \leftrightarrow_{df} \exists y\, x \in y$.

The basic idea is the same as before, it is just that $\in$ and Set are now stated explicitly to apply to logical objects only.

Mostly, I am going to focus on Definition 1, but for some purposes it will be useful to consider Definition 2 as an alternative, as will become clear in Section 4.

But why turn to epsilon terms at all and not use iota terms in these definitions? Indeed, following up Carnap's proposal, Lewis (1970) suggested to define theoretical terms in science by iota terms. And in the case of theories from empirical science, one might perhaps hope for these theories to describe the intended denotation of theoretical terms uniquely. However, in the case of a purely mathematical theory, any hope for uniqueness would be vain: for in all interesting cases, permutations of the underlying first-order domain would give rise to isomorphic but numerically distinct interpretations of the mathematical terms involved, which is why the uniqueness presupposition of iota terms would be violated. In contrast, non-uniqueness is unproblematic when mathematical concepts are defined by epsilon terms. What is more, the logicism I want to develop does not care about "the" intended interpretations of $\in$ and Set other than they satisfy $ZF2[R, S]$. Since $ZF2[R, S]$ is quasi-categorical (since $ZF2[\in, Set]$ is) and only includes logical expressions, one might also say: it only cares about the joint *logical structure* of $\in$ and Set (up to ordinal height). Structuralists about mathematics will concur,[19] though some non-structuralist realists about mathematics may not. But then again the task is to develop a logicist reconstruction of mathematics, not any such realist one.

There are further advantages to defining membership by a second-order epsilon term: assume that future set theorists will propose some new (say, large cardinal) axiom to be added coherently to $ZF2[\in, Set]$, and the mathematical community will go along with their proposal and regard the resulting system $ZF2^*[\in^*, Set^*]$ as their new foundation of mathematics. Then the "old" defining epsilon term $\epsilon R\, \exists S ZF2[R, S]$ from, say, Definition 1, could still be thought to denote the very same relation that a correspondingly updated logicism about $ZF2^*[\in^*, Set^*]$

[18] If one preferred, one could also define Set by yet another epsilon term, $Set =_{df} \epsilon X\, \forall x(X(x) \leftrightarrow \exists y\, x \in y)$, or, in this case (using second-order extensionality), even by $Set =_{df} \iota X\, \forall x(X(x) \leftrightarrow \exists y\, x \in y)$.

[19] See Boccuni and Woods (2020) for more on the affinity between (certain brands of) logicism and structuralism. See Leitgeb (2021) for the structuralist usage of epsilon terms to denote objects in ante rem structures: e.g., in an unlabeled graph G with two nodes and no edges, one may introduce a name a for one of these nodes by defining $a = \epsilon v(Vertex(v, G))$ and a name b for the other node by defining $b = \epsilon v(Vertex(v, G) \land v \neq a)$ (see Leitgeb, 2021, p. 79). a is then numerically distinct from b, but there is no non-semantic fact of the matter which of the two nodes is denoted by 'a' and which by 'b'. See Shapiro (2008, 2012) and Pettigrew (2008) for the closely related idea of regarding names for objects in structures, or for the structures themselves, as parameters introduced in the course of applying the logical rule of existential elimination. Schiemer and Gratzl (2016) also invoke epsilon terms in their reconstruction of structuralism.

would denote by its epsilon term $\epsilon R\,\exists S ZF2^*[R,S]$: although the equality

$$\epsilon R\,\exists S\,ZF2[R,S] = \epsilon R\,\exists S\,ZF2^*[R,S]$$

would not follow logically from $ZF2[\in, Set]$ and $ZF2^*[\in^*, Set^*]$, and even though there would be no fact of the matter whether that equality was true, it would be consistent with the two theories to accept it as true, and to speak as if membership as given by $ZF2[\in, Set]$ had always meant what would now be defined by means of $ZF2^*[\in^*, Set^*]$. It is that open-endedness of epsilon terms that Carnap aspired to exploit in his epsilon term reconstruction of theoretical terms, since he thought it nicely matched the open-endedness by which scientists may continue to specify the meanings of theoretical terms in the course of scientific development.[20] In the present context, it nicely matches the "inexhaustibility" of the concepts of set and membership that was described, e.g., by Gödel in his Gibbs lecture (Gödel, 1951 [1995]).[21]

Carnap's treatment of theoretical terms as epsilon terms may be viewed as a variant of his better known Ramsification reconstruction of a theoretical term T being given by a scientific theory $Th[T]$. He proposed to analyze $Th[T]$ in terms of what we now call the

Carnap sentence of $Th[T]$: $\exists R Th[R] \rightarrow Th[T]$

and the

Ramsey sentence of $Th[T]$: $\exists R Th[R]$.

The two of them taken together logically entail $Th[T]$ in second-order logic, and $Th[T]$ in turn logically entails their conjunction. Carnap's (1966) suggestion was to regard the Carnap sentence of $Th[T]$ as capturing the analytic content of $Th[T]$, since the only non-theoretical sentences (sentences without T) it entailed were logically true ones. In contradistinction, the Ramsey sentence of a typical theory $Th[T]$ from empirical science would capture the synthetic content of $Th[T]$, as it entailed the same non-theoretical (e.g. observation) sentences as $Th[T]$ itself.[22]

The correspondence to the epsilon term reconstruction of theoretical terms is: if T is defined by the epsilon term $\epsilon R Th[R]$, as suggested by Carnap (1959), the Carnap sentence of $Th[T]$ is indeed a logical consequence of that definition, which confirms its analytic status. Explained for the present context: since

(A) $\vdash \exists R \exists S\, ZF2[R,S] \rightarrow \exists S\, ZF2[\epsilon R\,\exists S\, ZF2[R,S], S]$

is an instance of the logical axiom scheme for second-order epsilon terms, 1 of Definition 1 combined with the Intersubstitutivity of Identicals yields the Frege-analyticity of

(B) $\exists R \exists S\, ZF2[R,S] \rightarrow \exists S\, ZF2[\in, S]$.

[20] "[. . .] this definition [by an epsilon term] gives just so much specification as we can give, and not more. We do not want to give more, because the meaning should be left unspecified in some respect, because otherwise the physicist could not—as he wants to—add tomorrow more and more postulates, and even more and more correspondence postulates, and thereby make the meaning of the same term more specific than it is today. So, it seems to me that the ϵ-operator is just exactly the tailor-made tool that we needed, in order to give an explicit definition, that, in spite of being explicit, does not determine the meaning completely, but just to that extent that it is needed" (Carnap, 1959, pp.171f).

[21] See Leitgeb (2023, Section 6) for more on the synchronic and diachronic advantages of dealing with semantic indeterminacy by means of epsilon terms.

[22] See Demopoulos (2007) and Suppl. E of Leitgeb and Carus (2024) for more on Carnap's reconstruction of theoretical terms in science.

And because $\forall x(S(x) \leftrightarrow \exists y\, x \in y)$ is logically derivable from (my formulation of) $ZF2[\in, S]$ in the deductive system of second-order logic, one can derive from (B) and 2 of Definition 1 the Carnap sentence

(C) $\exists R \exists S\, ZF2[R, S] \rightarrow ZF2[\in, Set]$,

which is thus Frege-analytic, too. Since Frege-analyticity will be seen to entail semantic analyticity in the next section, the Carnap sentence (C) of $ZF2[\in, Set]$ is therefore semantically analytic.

So far as the Ramsey sentence of $ZF2[\in, Set]$ is concerned, that is,

(R) $\exists R \exists S\, ZF2[R, S]$,

Sections 3 and 4 taken together will argue it to be likely to be semantically analytic, too, unlike the synthetic Ramsey sentences of typical empirical theories. And since semantic analyticity will be closed under logical derivability—and hence $ZF2[\in, Set]$ will be semantically analytic if (C) and (R) are—it will follow that $ZF2[\in, Set]$ is likely to be semantically analytic, just as promised. The same considerations apply *mutatis mutandis* to Definition 2 and its correspondingly expanded Carnap and Ramsey sentence.

But before I turn to the semantic analyticity of the Ramsey sentence (R), I will argue for the following: (i) Both Definition 1 and 2 indeed yield thesis 1d($\mathcal{L}^2_{\in,Set}$) from Section 1 for the language $\mathcal{L}^2_{\in,Set}$ of $ZF2[\in, Set]$. (ii) Even independently of the forthcoming argument for the analyticity of (R), Definition 1 (and analogously Definition 2) just by itself already amounts to a decent form of logicism, even when it does not quite deliver thesis 2d($ZF2[\in, Set]$) from Section 1.

About (i): given Definition 1, there are strong arguments in favor of 1d($\mathcal{L}^2_{\in,Set}$). For the only potentially non-logical terms in $\mathcal{L}^2_{\in,Set}$ are $\in$ and Set, and both of them are defined explicitly by purely logical terms: $\in$ is defined by the epsilon term $\epsilon R\, \exists S\, ZF2[R, S]$, which consists of only logical symbols, and since $\in$ is defined logically, the same holds for Set which is defined explicitly from $\in$. If there were a possible point of contention at all, it would concern whether the epsilon operator ϵ should count as logical. But see Woods (2014) for an argument to the effect that ϵ is logical in the Tarskian sense of permutation-invariance. Indeed, ϵ is very closely related to the existential and the universal quantifier: one can contextually define $\exists$ and $\forall$ from ϵ.[23] And, at the same time, the Second Epsilon Theorem (see Avigad and Zach, 2024) shows that the epsilon calculus is conservative over first-order logic: an epsilon-term-free first-order formula A is derivable in the epsilon calculus from a set Γ of epsilon-term-free first-order formulas just in case A is derivable from Γ in first-order logic. In that sense, the principles governing ϵ do not seem logically stronger than the logical axioms governing $\forall$ and $\exists$. Note that if it had been assumed that there was always a fact of the matter of what gets denoted by an epsilon term—and hence certain interpretations of ϵ would have been excluded as unintended on non-logical grounds—ϵ would have to be viewed as descriptive rather than logical; but, as explained before, this is not the case here.[24]

[23] E.g., in the first-order case: $\exists x \varphi[x] \leftrightarrow_{df} \varphi[\epsilon x\, \varphi[x]]$ and $\forall x \varphi[x] \leftrightarrow_{df} \varphi[\epsilon x\, \neg\varphi[x]]$. See Avigad and Zach (2024) for further details.
[24] Similarly, and for related reasons, the second-order Axiom of Choice had been regarded as logical, too.

Whether one is willing to come to the same verdict concerning $1\mathrm{d}(\mathcal{L}^2_{\in,Set})$ on the basis of Definition 2, too, depends of course on whether one is willing to grant logicality to $Logical\text{-}in\text{-}\mathfrak{C}$. But if there are logical objects, then a concept by which they can be qualified as such should at least count as logical in a slightly extended sense of (*meta*-)logicality. Compare: just as the truth values $true$ and $false$ are logical objects, the concept *truth value* that applies to them should count as a logical concept. In the case of logically true sentences or propositions, the concept of *logical truth* that characterizes them would generally be regarded as a logical concept in a similarly extended sense. And in provability logic, the provability of a logical truth ($Prov(\top)$) is expressed by the same logical operator by which the provability of that provability claim is expressed ($Prov(Prov(\top))$). In the same vein, the concept $Logical\text{-}in\text{-}\mathfrak{C}$ should qualify as a logical concept as well, and hence both Definitions 1 and 2 define $\in$ and Set in properly logical terms.

About (ii): As shown before, the Carnap sentence

(C) $\exists R \exists S\, ZF2[R, S] \rightarrow ZF2[\in, Set]$,

follows logically from Definition 1 in the deductive system of second-order logic extended by the epsilon calculus. Now consider any standard theorem of pure mathematics, such as, say, the Fundamental Theorem of the Calculus (FTC): FTC is known to be logically derivable from $ZF2[\in, Set]$ and suitable explicit set-theoretic definitions (such as of 'real number', 'real function', 'continuous', 'integral', and the like). By the Deduction Theorem, one can therefore logically derive

(D) $ZF2[\in, Set] \rightarrow FTC$

from these definitions. Consequently, if these definitions are combined with Definition 1, one can logically derive from that combination of definitions the sentence

(E) $\exists R \exists S\, ZF2[R, S] \rightarrow FTC$,

as (E) is logically derivable from (C) and (D). This means: even though the standard theorems of pure mathematics, such as FCT, are not quite logically derivable from explicit definitions themselves, one might still say that they are derivable from explicit definitions "in conditional form", that is, as consequents of conditionals in which the Ramsey sentence (R) serves as the antecedent.

In that sense, (quasi-)Carnapian logicism based on Definition 1 (or Definition 2) alone already amounts to a logicist variant of "if-thenism" or deductivism about mathematics,[25] as advocated e.g. in Russell's *Principles of Mathematics* (Russell, 1903). Moreover, even though it is sometimes claimed that Whitehead and Russell's *Principia Mathematica* (Whitehead and Russell, 1910–1910 1913) relied on the Axioms of Choice (or the Multiplicative Axiom) and the Axiom of Infinity—which were of questionable logicist status—what Whitehead and Russell actually suggested was to use these axioms as antecedents of conditionals, such that these conditionals would then be logically derivable in their ramified theory of types (see Whitehead and Russell, 1910–1910 1913, vol. 2, p. 183). The same strategy is employed by Carnap in his *Abriß der Logistik*,

[25] See Paseau and Pregel (2023) for a survey of deductivism.

in which he uses the Axiom of Choice and the Axiom of Infinity as antecedents of logically derivable theorems of simply type theory (see Carnap, 1929, Sections 24b and 24e). In the same manner, one might view the Ramsey sentence (R) as an "Extended Axiom of Infinity"[26] on the condition of which the standard theorems of pure mathematics become logically derivable in the deductive system of second-order logic extended by explicit definitions.

In fact, one can even do a bit better. One might rationally reconstruct mathematical practice as if it were engaged in an all-encompassing conditional proof: let us assume mathematicians suppose the Ramsey sentence (R) with the aim of deriving (given logic and definitions) theorems of pure mathematics, such as FTC from before. Once that has been achieved, one would normally finish such a conditional proof by discharging the assumption and concluding the corresponding conditional, such as (E) above. But now assume that mathematicians never actually get around to discharge their Ramsey sentence assumption but rather continue to work on the (implicit) presupposition that it holds true.[27] If viewed in this way, the proof patterns of actual mathematicians can be rationally reconstructed based solely on the extremely thin logicist grounds of Definition 1 or 2.

I hope this makes transparent why Definitions 1 and 2 are highly attractive for logicist purposes. However, one can still do better: for the Ramsey sentence (R) is not just any old presumption that mathematicians might want to make but may itself be seen to be (likely to be) semantically analytic in a suitable logicist framework. The required notion of semantic analyticity and the relevant logicist framework will be the topic of the next section.

3. Frameworks, Semantic Analyticity, and the Logicist Framework

It is time to shift our attention to the Ramsey sentence of $ZF2[\in, Set]$, that is,

(R) $\exists R \exists S\, ZF2[R, S]$.

Clearly, (R) only consists of logical symbols. If (R) is true, this means it is both a logical sentence and true, which, however, would not mean that (R) is *logically true*. In fact, given our standard Tarskian model-theoretic understanding of logical truth, (R) is of course not logically true, as there are second-order countermodels (e.g. all models with a finite domain). What I want to argue for in the following is that it is nevertheless analytic(ally true) in a suitably defined logicist framework. In contrast with more traditional conceptions of analyticity, such as Kant's, the required Carnapian concept of semantic analyticty-in-a-framework will allow for existence statements to be analytic, about which Carnap is perfectly explicit:[28] e.g., in Carnap (1950), he states that in a suitable arithmetical framework the existence of natural numbers and of prime numbers greater than a million are analytic, of which the former existence claim is trivial while the second one is less so. He also points out that classical logic comes with existence assumptions concerning individual constants such as '5', which may belong to the vocabulary of such a framework; if so, $\exists x\, x = 5$ is analytic because logically true.[29]

26 Indeed, it is easy to see that if (R) is satisfied by a full second-order model with a first-order domain of a certain cardinality, it is satisfied by every full second-order model with a first-order domain of a greater cardinality. In that sense, if (R) is satisfiable at all, it merely amounts to the claim that there are sufficiently many individuals.

27 This will match how I am going to describe the attitude of ordinary mathematicians towards (R) in Section 4.

28 See Ebbs (2017, Chapter 2) for further discussion.

29 Accordingly, there are possibility formulas in Carnap's (1946) modal predicate logic that are logically true, such as formulas of the form $\Diamond A$, in which A is a contingent non-modal sentence (see Carnap, 1946, p.64). Possibility formulas are the modal counterparts of existence formulas.

But of course it is one thing to acknowledge that Carnap accepted the analyticity of existence statements in certain frameworks and yet another to understand why this might make good philosophical sense. The philosophical point behind this is that conceptual frameworks are meant to organize information by structuring it in a particular manner—information that will then become expressible linguistically by sentences of the thereby interpreted object language of the framework. In that respect, Carnapian frameworks take over some of the structuring roles that space and time had for Kant,[30] though subject to some crucial differences: Kantian intuition of space and time is replaced by the linguistic expression of concepts and propositions; frameworks can be constructed and revised in a great plurality of ways, whilst Kantian space and time are simply given and unrevisable; and a Carnapian conceptual framework does not pre-determine what the empirical world is like, while for Kant e.g. the space of empirical intuition just *is* physical space. As a logical *empiricist*, Carnap regarded it to be the task of empirical science to find out, by observation and experiment, whether a contingent empirical sentence A or its negation $\neg A$ is true of the empirical world. But as a *logical* empiricist, he also thought that A and $\neg A$ are meaningful, and hence they—and with them the rest of their underlying language — come with some abstract conceptual and propositional structure that is constitutive of having thoughts about the world, independently of whether A or $\neg A$ is true of it. This structure needs to be in place prior to empirical investigation, and, once rationally reconstructed, it is that structure that a formal conceptual framework provides and assigns as interpretation to the sentences of its object language. Thereby, a conceptual framework comes itself with an ontological commitment to abstract structured thought. Carnapian logicism suggests to semantically interpret mathematics as dealing precisely with these abstract structured thoughts provided by the conceptual framework itself. The resulting interpretation may even extend to all of standard mathematics as we know it, if only the framework is complex enough, that is, if it provides sufficiently complex relational concepts. In the case of the logicist framework to be introduced below, the Ramsey sentence (R), that is, $\exists R \exists S\, ZF2[R, S]$, is going to express the ontological commitment to such a concept. And the analyticity of (R) in the framework will express that the ontological commitment is provided by the framework itself. Consequently, so long as information about the empirical world is structured according to the rules of the logicist framework to be introduced, each of the trivial classical logical law $A \vee \neg A$, the less trivial definition of membership in Definition 1, and the highly non-trivial Ramsey sentence $\exists R \exists S\, ZF2[R, S]$ will turn out to be (likely to be) true on purely conceptual grounds, independently of what the empirical world is like. In other words: they will be (likely to be) semantically analytic in the framework.[31]

The corresponding Carnapian concept of analyticity-in-a-framework is neither metaphysical nor epistemic in the sense of Boghossian (1996)[32] but rather semantic[33] in exactly the same sense in which Tarski's concept of truth is semantic. In fact, Carnap's definition of analyticity for Language II in his *Logical Syntax* amounts to an early version of a Tarskian definition of

[30] For more on this concerning Kant and time, see Sattig (2025), and for more on the general idea in the context of Carnap's *Aufbau*, see Richardson (1998).

[31] Other than its explicitly semantic formulation, this conception of mathematics is already present in Carnap's *Logical Syntax*. As Friedman (1999, p. 87) formulates it in his "Logical Truth and Analyticity in Carnap's 'Logical Syntax of Language' ": "Mathematics is built in to the very structure of thought and language and is thereby forever distinguished from merely empirical truth."

[32] Metaphysical analyticity is explained in terms of grounding or truthmaking, epistemic analyticity in terms of justification and cognitive grasp of meaning.

[33] I am in agreement with Lavers (2024, p.39) on this point. Otherwise, Lavers' (2024) understanding of, and argument for, the analyticity of large parts of mathematics differ very much from mine. (Lavers' idea is to determine the set of analytic sentences from statements and rules that emerge from the first stage of a Quinean explication.)

truth (see Suppl. G of Leitgeb and Carus, 2024), and once Carnap had fully embraced Tarskian semantics, he presented analyticity by reference to Tarskian semantic rules from the start:

> A sentence S_i is *L-true* in a semantical system S if and only if S_i is true in S in such a way that its truth can be established on the basis of the semantical rules of the system S alone, without any reference to (extra-linguistic) facts (Carnap, 1956, p.10),

where L-truth explicates analyticity, and where a semantical system is nothing but a conceptual framework in our terminology. And just as metaphysical necessity may be described as truth in all metaphysically possible worlds, that is, in all worlds in which the metaphysical laws are held fixed, analyticity-in-a-framework may also be described as truth in all worlds that are semantically possible in the relevant framework, that is, in all worlds in which the semantic rules of the framework are held fixed.

In Carnap's words:

> A sentence S_i is *L-true* (in S_I) $=_{Df}$ S_i holds in every state-description (in S_I) (Carnap, 1956, p.10)

and

> A sentence S is *A-true* in $L =_{df} S$ holds in all admissible models (Carnap, 1963, p. 901)

where A-truth explicates analyticity again.

It is important to note that this notion of analyticity is framework-relative (hence the "*L-true* (in S_I)" and "*A-true* in L"): much as the definition of a mathematical term may differ from one textbook to the next, since different textbooks may organize even the same body of mathematical knowledge differently, a sentence that is analytic in one conceptual framework may well fail to be analytic in another one. That is because the semantic rules of the frameworks may differ, and accordingly the class of semantically possible worlds in one framework may differ from the class of semantically possible worlds of another. Since analyticity in the present Carnapian sense is explicitly defined for, and relative to, constructed artificial frameworks, it is to be distinguished from the notion of analyticity in natural language that was mostly in the forefront of Quine's criticism in "Two Dogmas of Empiricism" (Quine, 1951). But I will not be able to enter the classical Carnap-Quine debate on analyticity here in any more detail.[34] Furthermore, the semantic notion of analyticity in a conceptual framework should be distinguished from metaphysical necessity, too. E.g., if metaphysical necessity got explicated in a conceptual framework with the help of a suitably constructed accessibility relation between worlds, every sentence that is analytically true in the framework would be metaphysically necessary but not necessarily the other way around.[35]

[34] For more on the debate, see Suppl. B of Leitgeb and Carus (2024).

[35] E.g., following Kripkean considerations, the accessibility relation might be constructed in a framework such that there is a semantically possible world at which $Son(a,b) \wedge \neg\Box Son(a,b)$ is true but where there is no metaphysically possible world at which that sentence is true. The reason for constructing a framework like that might be to rationally reconstruct the thought that the sentence does not invalidate any semantic rule but does invalidate the metaphysical necessity of hereditary relationships. ($\Box$ is meant to express metaphysical necessity, and $Son(a,b)$ is meant to express that a is son of b.)

Although Carnap did not use my term 'semantically possible world', he did evaluate formulas relative to entities that represented possible ways the world might be like (possible worlds, possibilities, possible cases, possible states of affairs), such that no semantic rule of the conceptual framework in question would be invalidated by that evaluation.[36]

In particular, Carnap (1942) states general postulates for the notion of the so-called L-range of a formula, by which Carnap's explicates the intensional meaning of a formula, that is, its truth conditions. In §18, he shows how these postulates can be realized by means of different procedures (A, B, C) that define, in a non-extensional metalanguage, L-ranges as classes of propositional entities (so-called L-states). In §19, he does the same for procedures that define L-ranges as classes in an extensional metalanguage: classes of (maximal) state descriptions (procedure E), classes of sentences (procedures F and G), and classes of so-called state-relations (procedures K and L). The state-relations of procedures K and L are similar to models (structures, interpretations) in contemporary model-theory in the sense that they are structured entities of objects and extensional properties/relations of these objects that can then be used to interpret and evaluate sentences.[37] Procedure E is applied later in his *Meaning and Necessity* (Carnap, 1947/1956) in which he presents formulas as holding at state descriptions, such that the (L-)range of a formula is the class of state-descriptions at which the formula holds (Carnap 1947/1956, p. 9). Furthermore, a formula is said to be true simpliciter just in case it holds at the actual state description (Carnap, 1947/1956, p.10); the same idea had been put forward in Carnap (1942, D18-B9) in terms of "rs", that is, "the real L-state". Clearly, this amounts to a precursor of present-day possible worlds semantics in which formulas are evaluated at worlds, one of which is regarded as actual. And in his later work (such as in Carnap, 1963 cited above or in Carnap, 1971), Carnap ends up evaluating formulas relative to models in the contemporary model-theoretic sense.[38]

My notion of semantically possible world in a conceptual framework is but a further development and application of these Carnapian ideas about semantics. So far as the metalanguage is concerned in which I will describe semantically possible worlds and the evaluation of formulas relative to them, I will follow Carnap's semantic work from the 1940s and use a language of higher-order logic instead of first-order set theory. It will be sufficient for my purposes to only sketch that higher-order language and the semantic rules that are formulated within it. The situation will resemble that of a typical logic textbook in which an object language—say, some second-order language—is specified in full formal detail, whereas the metalanguage in which the semantic rules for that object language are formulated remains partially unspecified (although a full formal specification could be given in principle).

Now let me turn to the logicist conceptual framework $\mathfrak{C}$, which involves the following components:

36 See Suppl. F of Leitgeb and Carus (2024) for more on Carnap's intensional semantics.

37 There are also differences: unlike models of modern model theory, which assigns e.g. a class of objects to each unary *predicate* of the object language, a state-relation in Carnap's procedure L assigns a class of objects to each extensional *property* that is to be expressed in the object language. Moreover, where modern model theory describes models in the language of standard first-order set theory, Carnap (1942) describe state-relations in the language of higher-order logic (type theory).

38 The fact that *Meaning and Necessity* (Carnap, 1947/1956) presented (L-)ranges as classes of state descriptions, and thus of syntactic entities, is sometimes interpreted as if Carnap had not left behind the syntactic emphasis of his *Logical Syntax* and hence had not fully embraced possible worlds semantics as yet. But that would be a misinterpretation: as he explains in Footnote 9 on p. 9 of Carnap (1947/1956), he only opted for applying procedure E from *Introduction to Semantics* because it seemed "the most convenient" one for the purposes of *Meaning and Necessity*. But other than that he might just as well have opted for a non-syntactic reconstruction of possible worlds, as witnessed by procedures K and L in Carnap (1942). I am grateful to Pierre Wagner for a discussion of these points.

(i) a second-order object language $\mathcal{L}$ with the usual primitive logical symbols of second-order logic, the additional primitive logical symbols ϵ and *Logical-in-*$\mathfrak{C}$, the defined predicates $\in$ and Set, and (for merely illustrative purposes) the primitive descriptive unary predicates 'Man' and '$Married$', and the defined unary predicate '$Bachelor$' (the last three predicates are tacitly relativized to a fixed point of time);

(ii) semantic rules for $\mathcal{L}$, formulated in a metalanguage of (cumulative) higher-order logic with ϵ, the logical predicate *Logical-in-*$\mathfrak{C}$, syntactic terms concerning the syntax of $\mathcal{L}$, the primitive descriptive unary predicates 'Man' and '$Married$', and some optional further expressions to be described in Section 4; the axioms and rules of a suitable deductive system of higher-order logic with extensionality and the epsilon calculus governing that metalanguage; and some further postulates, such as the definitions to be presented below and some optional additional postulates to be described in Section 4;

(iii) a class $\mathfrak{W}$ of models $\mathfrak{M}$, such that (iii.i) every way of assigning extensions to the primitive descriptive terms of $\mathcal{L}$ is realized by a (uniquely determined) $\mathfrak{M}$ in $\mathfrak{W}$, and (iii.ii) truth in each of these models $\mathfrak{M}$ respects the semantic rules for $\mathcal{L}$ in (ii).

(ii) means that $\mathfrak{C}$ involves some metalinguistic deductive components, whilst (iii) means that it also includes semantic components.

$\mathfrak{W}$ is of course the class of all semantically possible worlds of the framework $\mathfrak{C}$, which results from running through all combinatorial possibilities of assigning extensions to the primitive descriptive expressions in $\mathcal{L}$. Since all combinatorial possibilities are realized in $\mathfrak{W}$, it will be guaranteed that one of the worlds in $\mathfrak{W}$ corresponds to the actual world: it is the world at which, e.g., the extension of $Married$ is indeed the class of married humans at the fixed point of time; etc. (See the definition below.)

In contrast, a (proper) theory in $\mathfrak{C}$ would be given semantically by a proper subclass of $\mathfrak{W}$. Thus, unlike $\mathfrak{W}$, theories in $\mathfrak{C}$ rule out at least one semantically possible world in $\mathfrak{C}$, which is also why they are not guaranteed to include the actual world.

Finally, note that the semantically possible worlds of $\mathfrak{C}$ do not have any world-relative first-order or second-order domains assigned to them.

I will not state all of the semantic rules of $\mathfrak{C}$ for $\mathcal{L}$, but they include:

For all $\mathfrak{M}$ in $\mathfrak{W}$, for all variable assignments s:[39]

$Val_{\mathfrak{M},s}(Married(x)) = 1$ iff $\mathfrak{M}(Married)(s(x))$.
$Val_{\mathfrak{M},s}(Man(x)) = 1$ iff $\mathfrak{M}(Man)(s(x))$.
$Val_{\mathfrak{M},s}(Bachelor(x)) = 1$ iff not $\mathfrak{M}(Married)(s(x))$ and $\mathfrak{M}(Man)(s(x))$.
$Val_{\mathfrak{M},s}(\textit{Logical-in-}\mathfrak{C}(x)) = 1$ iff $\textit{Logical-in-}\mathfrak{C}(s(x))$.
$Val_{\mathfrak{M},s}(x \in y) = 1$ iff $Val_{\mathfrak{M},s}(\epsilon R \exists S\, ZF2[R,S])(s(x), s(y))$.
[$Val_{\mathfrak{M},s}(x \in y) = 1$ iff $Val_{\mathfrak{M},s}(\epsilon R \exists S(\forall z(S(z) \to \textit{Logical-in-}\mathfrak{C}(x)) \wedge ZF2[R,S])(s(x), s(y))$.]
$Val_{\mathfrak{M},s}(Set(x)) = 1$ iff $Val_{\mathfrak{M},s}(\exists y\, x \in y) = 1$.
$Val_{\mathfrak{M},s}(S(x)) = 1$ iff $s(S)(s(x))$.
$Val_{\mathfrak{M},s}(R(x,y)) = 1$ iff $s(R)(s(x), s(y))$.
$Val_{\mathfrak{M},s}(\neg\varphi) = 1$ iff not $Val_{\mathfrak{M},s}(\varphi) = 1$.

[39] In the present context, any talk of quantification over variable assignments s is short for: talk of second-order quantification over functions that map first-order variables to individuals, and talk of third-order quantification over functions that map second-order variables to second-order entities.

$Val_{\mathfrak{M},s}(\varphi \wedge \psi) = 1$ iff $Val_{\mathfrak{M},s}(\varphi) = 1$ and $Val_{\mathfrak{M},s}(\psi) = 1$.
$Val_{\mathfrak{M},s}(\forall x\, \varphi) = 1$ iff for all x-alternatives s' of s it holds: $Val_{\mathfrak{M},s'}(\varphi) = 1$.
$Val_{\mathfrak{M},s}(\forall R\, \varphi) = 1$ iff for all R-alternatives s' of s it holds: $Val_{\mathfrak{M},s'}(\varphi) = 1$.
$Val_{\mathfrak{M},s}(\epsilon R\, \varphi) = \epsilon s'(s'$ is an R-alternative of s and $Val_{\mathfrak{M},s'}(\varphi) = 1)(R)$.

As usual, the semantic rules determine uniquely, for each $\mathfrak{M}$ and s, an evaluation function $Val_{\mathfrak{M},s}$ that maps formulas in $\mathcal{L}$ to truth values. The evaluation of the formula $Married(x)$ at $\mathfrak{M}$ depends on what worldly extension $\mathfrak{M}(Married)$ the world $\mathfrak{M}$ assigns to the primitive descriptive predicate $Married$; analogously for $Man(x)$. The semantic rule for $Bachelor(x)$ encodes the definition of the defined descriptive predicate $Bachelor$ as applying precisely to unmarried men; since $Bachelor$ is defined from $Married$ and Man in $\mathfrak{C}$, its world-relative extension varies with those of $Married$ and Man. The semantic rules for the object-linguistic formula $Logical\text{-}in\text{-}\mathfrak{C}(x)$ invokes the meta-linguistic formula $Logical\text{-}in\text{-}\mathfrak{C}(x)$ (much as the semantic rule for $\neg$ involves 'not'). The semantic rules for $\in$ and Set encode Definition 1 (or Definition 2) from Section 2. The semantic rules for atomic formulas with a class variable S or a relation variable R are standard, as are those for the usual logical symbols. Finally, the semantic rule for object-linguistic epsilon terms $\epsilon R\, \varphi$ employs a metalinguistic epsilon term of the form $\epsilon s'(\ldots)$ in which s' is a variable for functions.

On that basis, we can define various further semantic notions well-known from intensional semantics: e.g., the proposition expressed by a sentence φ of $\mathcal{L}$ in $\mathfrak{C}$ is the class of semantically possible worlds $\mathfrak{M}$ in $\mathfrak{W}$, such that for all s, $Val_{\mathfrak{M},s}(\varphi) = 1$. The concept expressed by the unary predicate $Married$ of $\mathcal{L}$ in $\mathfrak{C}$ is the function that maps each world $\mathfrak{M}$ in $\mathfrak{W}$ to the extension $\mathfrak{M}(Married)$ at $\mathfrak{M}$. Etc.

Moreover, we can define actuality(-in-$\mathfrak{C}$), the metalinguistic semantic predicate 'true(-in-$\mathfrak{C}$)', and the metalinguistic semantic predicate 'analytic(-in-$\mathfrak{C}$)'. A world is actual just in case it assigns the "right" or intended extensions to all primitive descriptive predicates of the object language $\mathcal{L}$, as can be captured by translating these predicates into the metalanguage. Truth (simpliciter) of a sentence in $\mathcal{L}$ is its truth at the actual world (one can prove there is only one), while a sentence in $\mathcal{L}$ is analytic just in case it holds at all semantically possible worlds in $\mathfrak{C}$:

Metadefinition 3. (Actuality-in-$\mathfrak{C}$)

For all $\mathfrak{M}$ in $\mathfrak{W}$:

$\mathfrak{M}$ is actual(-in-$\mathfrak{C}$) iff for all d:
$d \in \mathfrak{M}(Married)$ iff d is married (at the given fixed point in time), and
$d \in \mathfrak{M}(Man)$ iff d is a man (at the given fixed point in time).

Metadefinition 4. (Truth-in-$\mathfrak{C}$)

For all sentences φ in the object language $\mathcal{L}$ of $\mathfrak{C}$:

φ is true(-in-$\mathfrak{C}$) iff
for all $\mathfrak{M}$ in $\mathfrak{W}$, for all s: if $\mathfrak{M}$ is actual(-in-$\mathfrak{C}$), then $Val_{\mathfrak{M},s}(\varphi) = 1$.

Metadefinition 5. (Analyticity-in-$\mathfrak{C}$)

For all sentences φ in the object language $\mathcal{L}$ of $\mathfrak{C}$:

φ is analytic(-in-$\mathfrak{C}$) iff for all $\mathfrak{M}$ in $\mathfrak{W}$, for all s: $Val_{\mathfrak{M},s}(\varphi) = 1$.

Hence, if a sentence of $\mathcal{L}$ is analytic(-in-$\mathfrak{C}$), it neither rules out any assignment of extensions to primitive descriptive terms in $\mathcal{L}$ nor any theory in $\mathfrak{C}$. For the same reason, an analytic sentence in the framework is guaranteed to be true at the actual world of the framework.

Here are some analytic example sentences in $\mathcal{L}$, the analyticity of which can be logically derived from the semantic rules of $\mathfrak{C}$:

$\forall x(Married(x) \vee \neg Married(x))$ is analytic(-in-$\mathfrak{C}$).
$\forall x(Bachelor(x) \leftrightarrow \neg Married(x) \wedge Man(x))$ is analytic(-in-$\mathfrak{C}$).
$\forall x, y(x \in y \leftrightarrow (\epsilon R \exists S\, ZF2[R, S])(x, y))$ is analytic(-in-$\mathfrak{C}$).
$\forall x(Set(x) \leftrightarrow \exists y\, x \in y)$ is analytic(-in-$\mathfrak{C}$).
$\exists R \exists S\, ZF2[R, S] \rightarrow \exists S\, ZF2[\epsilon R \exists S\, ZF2[R, S], S]$ is analytic(-in-$\mathfrak{C}$).

Thus, e.g., both parts of Definition 1 from Section 2 reappear as object-linguistic statements in $\mathcal{L}$ that are analytic(-in-$\mathfrak{C}$). The same applies to Definition 2 if the corresponding alternative semantic rule for $x \in y$ from above is used.

More generally, all logical axioms of the deductive system of second-order logic formulated in the object language are semantically analytic, and the same holds for all explicit definitions formulated in the object language and for all axioms of the epsilon calculus in the object language. Since Metadefinition 3 also clearly implies that semantic analyticity is closed under logical derivability, all Frege-analytic sentences in $\mathfrak{C}$ are therefore semantically analytic in $\mathfrak{C}$, as promised.

However, this does not mean that *every* sentence in $\mathcal{L}$ is such that it is analytic or its negation is analytic in $\mathfrak{C}$. E.g.:

$\exists x\, Bachelor(x)$ is not analytic(-in-$\mathfrak{C}$).
$\neg\exists x\, Bachelor(x)$ is not analytic(-in-$\mathfrak{C}$).

The reason for this is that there are semantically possible worlds in $\mathfrak{C}$ at which the extension of Man is a subclass of the extension of $Married$ and hence there are no bachelors, and there are semantically possible worlds in $\mathfrak{C}$ at which this is not the case and so there are bachelors. Similarly, a sentence expressing that there are exactly 1000 bachelors would not be analytic in $\mathfrak{C}$, and its negation would not be analytic in the framework either. This is just as intended: the truth or falsity of these claims does not just depend on the framework but also on the empirical facts; that is: it does not just depend on how information is structured in the framework but also on what information the actual world provides. Accordingly, some theories in the framework are going to claim that there are exactly 1000 bachelors, others that there are not, and yet others are going to claim neither. It is a matter of empirical investigation to confirm or disconfirm such theories, but all of these theories would be formulated against the backdrop of the framework $\mathfrak{C}$. If the relevant point of time is right now, we know in fact on empirical grounds that there are not exactly 1000 bachelors, so any object language sentence saying so is true at the actual world. Moreover, if $Bachelor$ had not been defined as applying to all and only unmarried men but had been regarded as primitive in $\mathfrak{C}$, the extension of $Bachelor$ would have varied independently of those of $Married$ and Man in the corresponding alternative framework $\mathfrak{C}'$. Hence, $\forall x(Bachelor(x) \leftrightarrow \neg Married(x) \wedge Man(x))$ would *not* have been analytic(-in-$\mathfrak{C}'$), since information would have been organized differently in $\mathfrak{C}'$ than in $\mathfrak{C}$.

Now let us return to our Ramsey sentence (R), which is a member of both the object language $\mathcal{L}$ of $\mathfrak{C}$ and of the metalanguage of $\mathcal{L}$ (that metalanguage also belongs to $\mathfrak{C}$).

The semantic rules of $\mathfrak{C}$ yield for all $\mathfrak{M}$ in $\mathfrak{W}$ and for all s:

$Val_{\mathfrak{M},s}(\exists R \exists S\, ZF2[R,S]) = 1$ iff
there is an R/S-alternative s' of s, such that $Val_{\mathfrak{M},s'}(ZF2[R,S]) = 1$ iff
there are an R and an S, such that $ZF2[R,S]$.

Using this, it follows:

$\exists R \exists S\, ZF2[R,S]$ is analytic(-in-$\mathfrak{C}$) iff
for all $\mathfrak{M}$ in $\mathfrak{W}$, for all s: $Val_{\mathfrak{M},s}(\exists R \exists S\, ZF2[R,S]) = 1$ iff
there are an R and an S, such that $ZF2[R,S]$.

The analytic truth of (R) in $\mathfrak{C}$ therefore boils down to a satisfiability claim,[40] that is, to the existence of higher-order R and S satisfying $ZF2[R,S]$. This result is a consequence of the definition of analyticity(-in-$\mathfrak{C}$), the fact that (R) only includes logical symbols, and the semantic rules of $\mathfrak{C}$. In particular, the semantic interpretation of the logical symbols is the same at all worlds, and the semantic rules in $\mathfrak{C}$ for existence claims do not invoke world-relative domains that would restrict the range of existential quantifiers. That is why the reference to worlds $\mathfrak{M}$ has dropped out from the evaluation of (R) once the semantic clauses have been fully unpacked. The analyticity of (R) in $\mathfrak{C}$ therefore follows to consist in the metalinguistic translation of (R) being the case.

More generally, if a sentence φ in $\mathcal{L}$ only includes logical symbols, then for all $\mathfrak{M}$ in $\mathfrak{W}$ and for all s it holds:

if $Val_{\mathfrak{M},s}(\varphi) = 1$ then φ is analytic(-in-$\mathfrak{C}$), and
if $Val_{\mathfrak{M},s}(\varphi) = 0$ then $\neg\varphi$ is analytic(-in-$\mathfrak{C}$).

Consequently, every logical sentence φ is analytic(-in-$\mathfrak{C}$) or its negation $\neg\varphi$ is analytic(-in-$\mathfrak{C}$), which is just as what Carnap had proved for all closed logical formulas of his languages I and II of his *Logical Syntax* (see Carnap 1934/1937, Theorems 14.3 and 34e.11).[41] This does not mean, of course, that for all logical φ, either φ is derivable from the deductive components of the framework $\mathfrak{C}$ or its negation $\neg\varphi$ is; after all, analyticity has been defined semantically, not proof-theoretically. It only means that purely logical statements are such that, if true, they are analytically true, and if false, they are analytically false.

The semantic rules for quantification in $\mathfrak{C}$ may be viewed as either tacitly assigning for each type one and the same domain to the quantifiers in $\mathcal{L}$ at all worlds, or as interpreting the quantifiers in $\mathcal{L}$ unrestrictedly, that is, as quantifying over everything of the right type—everything there is of that type (as expressed by the corresponding metalinguistic universal and existential quantifier).[42]

Indeed, for much of his work, Carnap himself used a "one-domain assumption" (cf. Hintikka, 1991, but see also Schiemer, 2013), and quantification over "absolutely everything" has been shown to be coherent if the semantic rules are formulated using the resources of higher-order logic (Williamson, 2003, see). Moreover, Linsky and Zalta (1994) and Williamson (1998) have advocated the analogous usage of possible worlds semantics with a single universal first-order domain for the interpretation of metaphysical modalities.

40 This bears some similarity to Hilbert's views on mathematical truth and consistency: "if the arbitrarily given axioms do not contradict one another. . . then they are true and the things defined by the axioms exist" (Hilbert, 1899, p.39).
41 Thus, if R is false in $\mathfrak{C}$, it is analytically false in $\mathfrak{C}$, i.e., its negation is analytic in $\mathfrak{C}$.
42 But note that what there is does not necessarily exhaust what is metaphysically possible.

Either way, since the worlds in $\mathfrak{W}$ are meant to track variations in extensional interpretation and not variations of what exists, it should be fair enough not to vary the ranges of quantifiers with worlds. Even more importantly for present purposes, Carnapian tolerance should allow us to set up our logicist framework as we please, so long as it may still count as logicist. And the world-independent interpretation of quantifiers in $\mathfrak{C}$ certainly does not undermine any logicist tenets.

So where does this leave us with the analyticity of the object-linguistic Ramsey sentence (R) in $\mathcal{L}$? It leaves us with the follow-up question

(Q) Are there R and S, such that $ZF2[R, S]$?

which is formulated in the metalanguage of $\mathcal{L}$ that also belongs to our logicist framework $\mathfrak{C}$. As shown before, if the answer to (Q) is 'yes', (R) will be analytic (-in-$\mathfrak{C}$), hence $ZF2[\in, Set]$ will be analytic(-in-$\mathfrak{C}$), and thus also part 2d($ZF2[\in, Set]$) of our logicist thesis from Section 1 will be vindicated.

In the next section I am going to argue that the answer to (Q) is indeed likely to be 'yes', which is why $ZF2[\in, Set]$ is likely to be analytic in $\mathfrak{C}$.

4. The (Likely) Analyticity of the Ramsey Sentence

One way of settling question (Q) from the last section would be by brute force: one might simply assume the metalinguistic translation of the Ramsey sentence (R) to be included in the metalinguistic deductive components of our logicist framework $\mathfrak{C}$, by which the analyticity of the object-linguistic Ramsey sentence (R) in $\mathfrak{C}$ would become derivable in $\mathfrak{C}$.

While this might seem a bit like cheating, there would be nothing in principle wrong about doing so. This said, there are three reasons for which I am nevertheless not going to pursue that strategy: first, we are only searching for an answer to (Q), not a provable answer. Put another way: the mere existence of an R and S satisfying $ZF2[R, S]$ is sufficient for (R) being analytic(-in-$\mathfrak{C}$). Therefore, while *proving* that existence claim would conveniently deliver the existence of such R and S, it would also go beyond what is required.[43] Second, consider anyone who might still question (perhaps on Quinean holistic grounds) the viability of distinguishing between the conceptual framework $\mathfrak{C}$ and the proper theories in $\mathfrak{C}$, as presented in the last section: any such person would surely feel only more concerned if $\mathfrak{C}$ were to include deductively strong components, such as the metalinguistic translation of (R). And third, the stronger the deductive components of a conceptual framework, the greater the risk of the framework being inconsistent, and inconsistency would be just as unattractive to the constructor of a Carnapian framework based on classical logic as it would be to anyone putting forward a scientific theory based on classical logic. So I refrain from building (R) into the framework deductively: I will leave the deductive components of the framework $\mathfrak{C}$ as deductively weak as they were described in the last section, consisting just of semantic rules, a deductive system of logic, and explicit definitions.[44]

Instead, I suggest conducting the following little thought experiment: *what if one presented the conceptual framework $\mathfrak{C}$ from the last section to ordinary mathematicians and set theorists?* One would

[43] Compare the related discussion in Awodey and Carus (2003, 2004), who point out against Gödel that a Carnapian framework based on classical logic does not have to be *provably* consistent, just consistent.

[44] I am grateful to an anoymous reviewer for urging me to comment on this point.

explain to them that the quantifiers in (R) are meant to range over everything of the right type, or that there is a fixed intended universe of discourse that is tacitly meant to include all of the usual mathematical entities of the right type. And then one would pose to them question (Q) as a logical-mathematical question:

(Q) Are there R and S, such that $ZF2[R, S]$?

In their roles as experts for such logical-mathematical questions, *what would they answer?*

I take it that most ordinary mathematicians accept or presuppose $ZF2[\in, Set]$ as a coherent interpreted background language that has never led to contradictions and which they find more or less conducive to their own mathematical work—work that does not itself concern models of set theory but rather number-theoretic properties of integers, probabilistic properties of random walks in graphs, fixed-point properties of continuous functions on topological spaces, and the like. For that reason, they should be willing to accept or presuppose (R), too, as (R) is logically entailed by $ZF2[\in, Set]$ in the deductive system of second-order logic, and they have been willing to accept or presuppose $ZF2[\in, Set]$ as a foundation. If they were forced to comment more particularly on the existence of *set*-sized models of $ZF2[\in, Set]$ and hence to comment on the existence of *set* values of R and S in 'there are R and S, such that $ZF2[R, S]$' (rather than proper-class-sized entities), they might point out: no one knows conclusively whether such a set model exists, as it seems that we can neither derive (R) nor its negation from uncontroversial principles. After which they might defer to the experts on set models, that is, their set theorist colleagues.

In turn, set theorists do study models of set theory. And they do have more to say about the existence of models of $ZF2[\in, Set]$: they might put forward the established result that if there is a strongly inaccessible cardinal greater than ω, then there is a set model of $ZF2[\in, Set]$. And at least those set theorists (called "absolutist practitioners" in Kant, 2025,?) who believe in the existence of a uniquely determined universe of sets that makes certain set-theoretic axioms true would voice their belief in the existence of such strongly inaccessible cardinals.[45] And they might give arguments for this, too, even when these arguments could not be formally reconstructed as proofs in $ZF2[\in, Set]$ or first-order ZFC (assuming these theories to be consistent, as set theorists very strongly believe them to be).[46] So at least "absolutist" set theorists would not just answer (Q) with a 'yes', they would even think the witnesses to 'there are R and S, such that $ZF2[R, S]$' may be taken to be sets. Of course, they might still be wrong about all of that—after all, no deductively valid argument with obviously true premises has been put forward. But there still seem to be an *inductively strong arguments* (in the sense of Skyrms, 2000, p.17) in favor of (R): arguments that make (R) likely or plausible. Just as all other inductively strong arguments, they do not guarantee the truth of their conclusion given

[45] See Kant (2025, 81–3) and Kant (2025, 114), who examined this empirically, and who reports that absolutist practitioners believe in the truth of large cardinal axioms, at least up to Woodin cardinals (and thus including strongly inaccessible cardinals). Moreover, set theorists in general widely use large cardinal axioms (Kant, 2025, p. 110), and they believe large cardinal axioms are consistent (Kant, 2025, p.113). (I am very grateful to Deborah Kant for her help on this matter.) Džamonja (2017, Section 3) comments on large cardinals in a similar manner: "Not only are the large cardinals needed for set theory but they are also known to be needed for some seemingly innocent statements about number theory. For example, Harvey Friedman [. . .] developed the Boolean relation theory, which demonstrates the necessity of large cardinals for deriving certain propositions considered "concrete". Friedman and others view this as an obvious reason for a working mathematician to accept large cardinals."

[46] See Hrbacek and Jech (1999, pp.279f) for such an argument. Kant (2025) also makes the point that even set theorists who are finally interested in first-order ZFC proofs (such as in descriptive set theory) regularly use large cardinal axioms and then eliminate them in their proofs. This may be viewed as an argument for the thesis that the assumption of the existence of a strongly inaccessible cardinal is at least instrumentally acceptable for these set theorists.

their premises, but that does not mean that they do not supply any justification whatsoever, and arguing inductively may well be the best we can do at that foundational level.

Summing up: I think it is fair to say that what the verdicts of the experts—ordinary mathematicians and set-theorists—would reveal about their beliefs about (R) in our little thought experiment can be rationally reconstructed as a *high-probability assignment to* (R). Given that, it must be at least as likely that the Ramsey sentence (R) is analytic(-in-$\mathfrak{C}$). I am going to make this probabilistic reconstruction a bit more precise now. Afterwards, I will address two potential worries about the thought experiment.

So far as ordinary mathematicians are concerned, their mathematical statements may best be reconstructed as made *from within our framework* $\mathfrak{C}$ and hence as belonging to the object language $\mathcal{L}$ of $\mathfrak{C}$. The mathematicians' belief or acceptance of such statements may then be reconstructed by means of subjective probability measures that assign probabilities to the members of $\mathcal{L}$. Accordingly, in Carnap's work on inductive logic (see e.g. Carnap, 1950), a conceptual framework such as our $\mathfrak{C}$ is expanded by a corresponding class of such subjective probability measures—say, the class $Prob_{\mathfrak{C}}$—precisely for the purpose of capturing rational inductive reasoning that takes place internally to the framework. And what was said above about mathematicians generally accepting or presupposing $ZF2[\in, Set]$ and hence (R) will then correspond to: for all P in $Prob_{\mathfrak{C}}$ it holds that $P(R) = 1$. That is: for mathematicians speaking from within the framework it is not an epistemic possibility that (R) fails, since for them (R) is epistemically presupposed in their mathematical work and hence must be counted as (group-subjectively) probabilistically certain. If (R) is indeed analytic-in-$\mathfrak{C}$, this intended probabilistic reconstruction will automatically follow from (R) being true in every semantically possible world in $\mathfrak{C}$, and from the probability of a sentence A of the object language of $\mathfrak{C}$ corresponding to the probability of the class of semantically possible worlds of $\mathfrak{C}$ in which A is true (see Carnap, 1971).[47] However, for the same reason, we cannot extract much of an argument in favor of (R) from the ordinary mathematicians' verdicts about (R) other than they are willing to accept or presuppose (R) in their mathematical work.

Now for the rational reconstruction of what is conveyed by the set-theorists' verdicts: their statements may be reconstructed as belonging to the metalanguage of the object language $\mathcal{L}$ of our framework $\mathfrak{C}$, as they are reflecting on models of mathematics and set theory. The beliefs or acceptances that these statements express should thus be captured by subjective probability measures that assign probabilities not to the sentences of the object language $\mathcal{L}$ but of the metalanguage of $\mathcal{L}$ (the same language in which analyticity-for-$\mathcal{L}$ had been defined). Since we have seen set theorists would generally answer (Q) with a reasonably strong affirmation based on inductively strong plausibility arguments, their answer may be rationally reconstructed as expressing a high (group-subjective) probability claim of the form *it is likely that there are* R *and* S, *such that* $ZF2[R, S]$. And since it seems rational to defer to the experts on that subject matter, our own rational degrees of belief should concur.

On that basis, summarized in slightly compressed terms, we get the following informal and partially *probabilistic* metalinguistic argument for (quasi-)Carnapian logicism, in which '$P(A) = \ldots$' is a rational-degree-of-belief operator applicable to the sentences A of the metalanguage of $\mathcal{L}$

[47] Indeed, a sentence A in the object language $\mathcal{L}$ of framework $\mathfrak{C}$ might be defined to be *apriori relative to* $\mathfrak{C}$ just in case for all probability measures P in $Prob_{\mathfrak{C}}$ it holds that $P(A) = 1$. I regard this as a suitable rational reconstruction of the epistemic notion of relative or constitutive aprioricity discussed by Friedman (2001) (amongst others), but I will not defend this claim here. Note that every sentence that is analytic-in-$\mathfrak{C}$ is also apriori relative to $\mathfrak{C}$ but not necessarily the other way around.

in our logicist conceptual framework $\mathfrak{C}$, such that the respective subject whose rational degrees of belief are denoted by '$P(\cdot)$' is *us*.[48] '$Analytic$' is short for 'analytic(-in-$\mathfrak{C}$)', 'ε' denotes some small but only vaguely determined number, I will suppress all matters to do with quotation, and I will concentrate just on Definition 1 and (R) again:

(a) $(\in =_{df} \epsilon R \exists S\, ZF2[R,S]) \wedge \forall x(Set(x) \leftrightarrow_{df} \exists y\, x \in y)$.
(b) $P(Analytic((\in = \epsilon R \exists S\, ZF2[R,S]) \wedge \forall x(Set(x) \leftrightarrow_{df} \exists y\, x \in y))) = 1$.
(c) $P(Analytic(\exists R \exists S\, ZF2[R,S] \rightarrow \exists S\, ZF2[\epsilon R \exists S\, ZF2[R,S], S])) = 1$.
(d) $P(\exists R \exists S\, ZF2[R,S] \leftrightarrow Analytic(\exists R \exists S ZF2[R,S])) = 1$.
(e) $P(\exists R \exists S\, ZF2[R,S]) = 1 - \varepsilon$.
(f) $P(Analytic(\exists R \exists S\, ZF2[R,S])) = 1 - \varepsilon$.
(g) $P(Analytic(\exists S\, ZF2[\epsilon R \exists S\, ZF2[R,S], S])) \geq 1 - \varepsilon$.
(h) $P(Analytic(ZF2[\in, Set])) \geq 1 - \varepsilon$.
(i) $(\in =_{df} \epsilon R \exists S\, ZF2[R,S]) \wedge \forall x(Set(x) \leftrightarrow_{df} \exists y\, x \in y) \wedge\ P(Analytic(ZF2[\in, Set])) \geq 1 - \varepsilon$

Therefore, (quasi-)Carnapian logicism holds.

(a) is Definition 1 from Section 2, but now viewed as a metalinguistic statement that says correctly how the object-linguistic terms $\in$ and Set in $\mathcal{L}$ have been defined in $\mathfrak{C}$. Since Definition 1 is provably analytic in $\mathfrak{C}$, as shown in Section 3, (b) rightly states that the subjective probability that Definition 1 is analytic(-in-$\mathfrak{C}$) is 1 (since we are certain that (a) is the case). The same holds for the subjective probability of the analyticity of the Carnap sentence (C) in (c). (d) reflects it being provable in $\mathfrak{C}$ that the analyticity of (R) boils down to the metalinguistic translation of (R), as demonstrated in Section 3. (e) is the rational reconstruction of our deference to our set theorists' informed verdicts about that metalinguistic translation of (R). (f) follows from (d) and (e) by the axioms of probability. (g) follows from (c) and (f) together with the logical closure of analyticity (shown in Section 3 and us being certain of it) and the axioms of probability. Similarly, (h) follows from (g), (b), the logical closure of analyticity, and the axioms of probability. (i) just joins (a) and (h) by conjunction.

But (i) yields the promised thesis of (quasi-)Carnapian logicism from Section 1, since $\mathfrak{C}$ is a framework in which all mathematical terms in $\mathcal{L}^2_{\in,Set}$ are logical in $\mathfrak{C}$ (1d($\mathcal{L}^2_{\in,Set}$), and all mathematical theorems of $ZF2[\in, Set]$ are likely to be analytic in $\mathfrak{C}$ (2d($ZF2[\in, Set]$.

Let me conclude by addressing two potential worries about our previous little thought experiment of asking mathematicians and set theorists about (R)—one epistemological, the other one ontological. The epistemological one is: is it permissible for a logicist about mathematics to justify a statement by asking mathematicians for their opinion about it? Wouldn't that be viciously circular? And the ontological worry is: the reason set theorists believe there to be R and S that satisfy $ZF2[R,S]$ is that they strongly believe there to be a relation of *sets* and a class of *sets* that jointly satisfy $ZF2[R,S]$. But what reason do we have to believe these sets are logical objects, as one might perhaps require of a logicism about mathematics?

A brief inspection of the logicist thesis I promised to defend in Section 1 should swiftly clarify that it is neither epistemological nor ontological in nature but rather semantic: 1c was about terms and their meaning in a framework, whilst 2c was about theorems and their semantic

[48] Carnap (1950, 1971) would have rationally reconstructed such a probabilistic argument in yet another conceptual framework $\mathfrak{C}^*$, so that the metalanguage in $\mathfrak{C}$ would have become the object language $\mathcal{L}^*$ in $\mathfrak{C}^*$. And then he would have considered logical probability measures that would assign probabilities to the members of $\mathcal{L}^*$. Subjective probability measures such as P would have resulted from conditionalizing such logical probability measures on the available evidence. But I will not be able to go into any more detail on this. See Sznajder (2018) for more on Carnap on inductive probability.

analyticity in a framework. And the existence of a logicist framework in which 1c and 2c are the case was promised to follow from the existence of a logicist framework in which the claims 1d($\mathcal{L}^2_{\in,Set}$) and 2d($ZF2[\in, Set]$) are the case, which are equally semantic. Thus, neither the epistemological nor the ontological worry expressed before actually concerns the logicist project of this paper.

In particular, while e.g. Frege's logicism was certainly at least partially motivated by epistemological concerns, (quasi-)Carnapian logicism is not. Of course, the successful rational reconstruction of a scientific theory may occasionally improve the epistemic standing of that theory. But a logicism about mathematics that proceeds by logically reconstructing the axiomatic theory $ZF2[\in, Set]$, which may itself be viewed as having resulted from the set-theoretic rational reconstruction of mathematical practice, would be extremely unlikely to stand on better justified grounds than $ZF2[\in, Set]$ itself. And indeed none of this is the point of (quasi-)Carnapian logicism, and it has not been claimed to be so either. On the contrary, a (quasi-) Carnapian logicist may happily admit that logic and set theory are epistemologically on par, which is why asking our set theory experts for their advice on a higher-order existence statement should hardly count as a no-go.[49]

With respect to the ontological worry from above, (quasi-)Carnapian logicism is not affected by it because its logicist thesis only concerns the logicality of mathematical terms and the analyticity of mathematical theorems, not the logicality of mathematical objects. As mentioned in Section 2, its application to quasi-categorical second-order set theory only cares about logical structure, not what the entities are like that are structured as such. Accordingly, it does not matter whether the witnesses to 'there are R and S, such that $ZF2[R, S]$' are physical entities, mental entities, proper relations/classes of sets, or quite simply sets, so long as the object-linguistic Ramsey sentence (R) comes out as analytic(-in-$\mathfrak{C}$).

This said, one might also consider a variant of (quasi-)Carnapian logicism that would expand its focus beyond mathematical terms and theorems to *objects*: the corresponding extended logicist theses would still start with

> There is a logicist conceptual framework, such that [. . .]

but then extend 1c and 2c by

3c. all standard mathematical objects are logical objects in the framework,

and extend 1d($\mathcal{L}^2_{\in,Set}$) and 2d($ZF2[\in, Set]$) by

3d(Set). all members of Set are logical objects in the framework.

That is where Definition 2 from Section 2 comes in handy: assume the explicit epsilon term definition of $\in$ (and indirectly of Set) in $\mathfrak{C}$ to include the restriction to objects that are $Logical\text{-}in\text{-}\mathfrak{C}$. And consider the members of the class $Logical\text{-}in\text{-}\mathfrak{C}$ to be abstract objects introduced by the framework $\mathfrak{C}$ itself. This would be in line with how Carnap's (1950) "Empiricism, Semantics, and Ontology" describes what it takes for a framework to introduce a new class of abstract objects: the framework needs to provide a general term for these objects

[49] This deference to set theorists only pertains to the existence of R and S satisfying $ZF2[R, S]$, not to any philosophical thesis of logicism about mathematics. Set theorists are experts concerning the former but not concerning the latter.

($Logical\text{-}in\text{-}\mathfrak{C}$), expressions for properties or relations of these objects ($\in$), variables for them ($x, \ldots$), quantifiers that bind these variables ($\forall x, \exists x, \ldots$), and rules of formation and inference, including logical rules for the quantifiers (such as, e.g., universal instantiation). Clearly, all of these conditions are satisfied here.[50] Indeed, Carnap's "variables of the new type" for the abstract objects introduced by a framework may be regarded as expressing in the formal mode what contemporary abstractionists would express in the material mode by: "abstraction may result in 'new' objects'..." (Linnebo, 2018, p.55). While Carnap did not make the abstraction process underlying the introduction of a new class of abstract objects by a framework explicit, the resulting abstract objects may certainly be qualified as *thin* "in the sense that their existence does not make a substantial demand on the world" (Linnebo, 2018, p.xi). Formulated less metaphysically, one might say that the concept of existence that is employed when the existence of logical objects of the framework is postulated within the framework is just that expressed by the purely logical $\exists x(Logical\text{-}in\text{-}\mathfrak{C}(x) \wedge \ldots)$, which is logically independent of the existence or non-existence of men, married people, bachelors, or other non-abstract objects.[51]

Analogously to the case of (R) before, the corresponding Ramsey sentence

(R_{Log}) $\exists R\,\exists S(\forall x(S(x) \rightarrow Logical\text{-}in\text{-}\mathfrak{C}(x)) \wedge ZF2[R, S])$

follows to be analytic(-in-$\mathfrak{C}$) if and only if there are R and S, such that $\forall x(S(x) \rightarrow Logical\text{-}in\text{-}\mathfrak{C}(x)) \wedge ZF2[R, S]$. Hence, if there are such R and S, then it will not just be the case that standard pure mathematics is analytic(-in-$\mathfrak{C}$) but additionally $\in$ and Set—as defined in $\mathfrak{C}$—will apply to objects that are $Logical\text{-}in\text{-}\mathfrak{C}$. If so, even 3d($Set$) and consequently 3c will be satisfied in $\mathfrak{C}$.[52]

The only downside would be that the corresponding question

(Q_{Log}) Are there R and S, such that $\forall x(S(x) \rightarrow Logical\text{-}in\text{-}\mathfrak{C}(x)) \wedge ZF2[R, S]$?

could no longer be addressed just by asking ordinary mathematicians or set theorists. For ordinary mathematicians are experts for ordinary mathematical objects and set theorists have additional expertise on sets, but neither are experts for logical objects, let alone logical objects in $\mathfrak{C}$. However, this remaining gap can be bridged: first add the set-theorists' terms Set and $\in$ to the vocabularly of the metalanguage of $\mathcal{L}$ in $\mathfrak{C}$; and then extend the deductive metalinguistic components of $\mathfrak{C}$ by the metalinguistic higher-order assumption that the logical relation $Val_{\mathfrak{M}}(\in)$ structures the logical objects in $Val_{\mathfrak{M}}(Set)$ in the same manner in which the set-theoretic membership relation $\in$ structures sets. That is:

(Ass_{Log}) $\langle Val_{\mathfrak{M}}(Set), Val_{\mathfrak{M}}(\in)\rangle \cong \langle Set, \in\rangle$.

With that in place, the previous high probability of there being R and S such that $ZF2[R, S]$, which resulted from set-theoretic considerations about $\langle Set, \in\rangle$, translates immediately into a high probability for there being R and S, such that $\forall x(S(x) \rightarrow Logical\text{-}in\text{-}\mathfrak{C}(x)) \wedge ZF2[R, S]$,

[50] Carnap (1950a, Section 3) actually speaks of the introduction of variables of a "new type", which would presuppose a many-sorted logic. But instead of introducing a new dedicated class of variables, one may just as well use one sort of variables and restrict them by the new general term $Logical\text{-}in\text{-}\mathfrak{C}$ instead.

[51] See Suppl. H of Leitgeb and Carus (2024) for more on Carnap on ontology.

[52] More should be said about what makes the members of the class $Logical\text{-}in\text{-}\mathfrak{C}$ properly *logical* (rather than just abstract). The key to this, in my view, would to argue that the members of $Logical\text{-}in\text{-}\mathfrak{C}$ might be regarded as abstract meaning-entities (Fregean senses or Carnapian intensions). But I will leave this to one side here.

which is the metalinguistic translation of (R_{Log}). Therefore, even R_{Log} ends up very likely analytic(-in-$\mathfrak{C}$). And the additional assumption (Ass_{Log}) hardly adds to the deductive strength of $\mathfrak{C}$, as it merely states that logicist sets and ordinary sets are structured alike, without saying which such sets exist and what their structure is like.

5. Conclusions

I have argued for (quasi-)Carnapian logicism: there is a logicist conceptual framework in which $\in$ and Set are defined in logical terms, and in which $ZF2[\in, Set]$ is (likely to be) semantically analytic. It follows that all standard terms of pure mathematics are logical in the framework, and all standard proven theorems of pure mathematics are (likely to be) semantically analytic in the framework. The required definitions, the semantic notion of analyticity in a framework, the logicist framework, and the occurrence and justification of the probabilistic qualification "likely to be" have been explained in the previous sections.

The essential features of the resulting Carnapian brand of logicism are: it is clear, formally precise, systematic, and reasonably simple. It still resembles mathematical practice in so far as it preserves the usual set-theoretic definitions of mathematical terms, it preserves the set-theoretic proofs of mathematical theorems, it acknowledges the open-endedness of the concepts of sethood and membership, it makes the existential presupposition of the set-theoretic treatment of mathematics explicit, and it incorporates (hypothetical) verdicts and arguments by ordinary mathematicians and set-theorists into its argument for the likely analyticity of that presupposition within the logicist framework. Its upshot is that pure mathematics can be rationally reconstructed as purely conceptual in the sense of coming along with a conceptual framework, while staying close to mathematical practice.[53] As shown in Section 4, the 'purely conceptual' can be extended even to the ontology of mathematics, to the effect that all mathematical objects are logical objects in the respective logicist framework. Finally, Carnapian Logicism is embedded in, and coheres with, Carnap's understanding of logic, theoretical terms, conceptual frameworks, analyticity, and probability and matches his overall conception of philosophy as rational reconstruction.

What Carnapian logicism does *not* achieve (and does not aim to achieve), as has been explained in Section 4, too, is to give mathematics a secure logical foundation. Epistemologically, it remains on the same level as Frege's *Grundlagen* in which Frege points out:

> I do not claim to have made the analytic character of arithmetical propositions more than probable. . . (Frege, 1884, *Die Grundlagen der Arithmetik*, §90)

In Frege's case, that was because the *Grundlagen* had not quite delivered sound, formally precise, and gap-free logical derivations of the mathematical laws of arithmetic from axioms of logic. That is what he hoped to supply in his later *Grundgesetze*, though we now know that he would fail to do so. In the meantime, Gödel's Incompleteness Theorems have made it seem unlikely that *any* logicist could do better than arguing for the analyticity of mathematics on probabilistic grounds.

[53]In contrast, empirical science could not be rationally reconstructed as purely conceptual while staying close to scientific practice. But it is not the place to argue for this claim here.

There might be one other potential downside to Carnapian Logicism: consider e.g. the Continuum Hypothesis, which we know is neither provable nor refutable in $ZF2[\in, Set]$. It is well-known that the Continuum Hypothesis can be reformulated as a statement CH in the language of pure second-order logic, and the same holds for its negation, which can also be expressed as a statement NCH in the same language (see Shapiro, 1991, p.105). Moreover, it is easy to see that either CH is logically true in full (model-theoretically defined) second-order logic or NCH is logically true in full second-order logic. For the same reason, either CH is analytic in the logicist framework from Section 3 or NCH is analytic in that framework, even though we do not know which of the two is the case. This matches Bohnert's (1975, p.211) summary of what Carnap told him in 1967, that is, "one could only wait and watch developments, with respect to what could be thought of as analytically true", where in the present case 'analytically true' would not be relative to our pre-theoretic understanding of set and membership but to the understanding afforded by the logicist framework from Section 3. At the same time, if it happened to be the case that mathematicians did not think CH is "settled" in that manner, this would amount to an important discrepancy between our logicist rational reconstruction of mathematics and what mathematicians would think themselves.[54] On the other hand, rational reconstructions are merely required to be similar to what they reconstruct; certain discrepancies are to be expected. And of course the deductive system of the logicist framework of this paper does not settle the question by means of proof, which might be all these mathematicians might mean by the Continuum Hypothesis not being settled. I will have to leave this question to future work.

In any case: as things stand, the resulting Carnapian logicist package does not seem to fare worse than any other philosophical interpretation of mathematics available. It is a coherent option that is on offer for anyone willing to choose it.

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[54] I am grateful to an anonymous reviewer for highlighting this.

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