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Carl Auer von Welsbach (1858-1929) - A Famous Austrian Chemist Whose Services Have Been Forgotten for Modern Physics

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Abstract. Carl Auer von Welsbach (1858-1929) was a chemist and entrepreneur famous beyond the borders of Austria, with good contacts to well-known chemists and physicists in Europe. This is evidenced by his extensive correspondence and the shipment of his rare earth preparations to known researchers. In 1895 he discovered the elements neodymium and praseodymium and in 1905 the elements ytterbium and lutetium. In his time he was considered a specialist for the rare earth elements (REE). He received his doctorate from Robert Bunsen in Heidelberg (1880-1882). Spectral analysis was his domain. His ability to neatly separate the chemically very similar SE elements from the minerals (e.g. monazite sand) to the then-known and further developed principle of “fractional crystallization” also made relatively accurate investigations of the magnetic properties of these elements possible. In particular, the chemists and physicists were interested in the question of whether or not the series of REE elements is complete with lutetium. The famous quantum physicist Niels Bohr had made a statement with his atomic model that lutetium must be the last element of this sequence in the periodic table of the elements and predicted the magnetic properties. They were confirmed by the experiments with the Auer von Welsbach preparations - in particular of lutetium - by the physicist Stefan Meyer (1842-1949) in Vienna. In 1925 the physicist and theoretician Friedrich Hund (1886-1997) from Göttingen then succeeded to set up a first quantum mechanical model (Hund’s rule), which achieved good agreement with the experimental results from Vienna. This was an advance in early quantum mechanics, which is also due to the highly pure SE preparations of the chemist Carl Auer von Welsbach.

Keywords. Carl Auer Welsbach, Rare earth Elements, magnetic properties, Niels Bohr, Friedrich Hund, quantum mechanics.

1. INTRODUCTION. CARL AUER VON WELSBACH – THE MOST IMPORTANT STAGES OF HIS LIFE AS A RESEARCH SCIENTIST

Carl Auer von Welsbach was born on September 1st 1858 in Wels (Austria) and later especially known for the invention of the incandescent gas mantle. His father Alois Welsbach was already famous as chief of the imperial court printing house and well-known beyond Austrian borders.

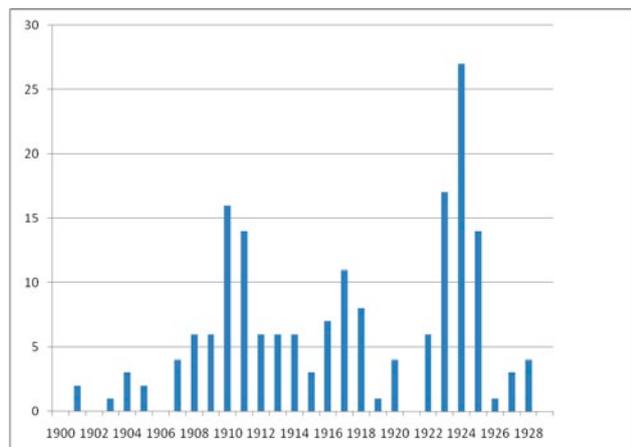


Figure 2. Auer von Welsbach – correspondence with chemists and physicists in Europe. Source¹⁴: Gerd Löffler, *Carl Auer von Welsbach und sein Beitrag zur frühen Radioaktivitätsforschung und Quantentheorie*, ISBN 978-3-200-04400-5, 2015, p. 127,

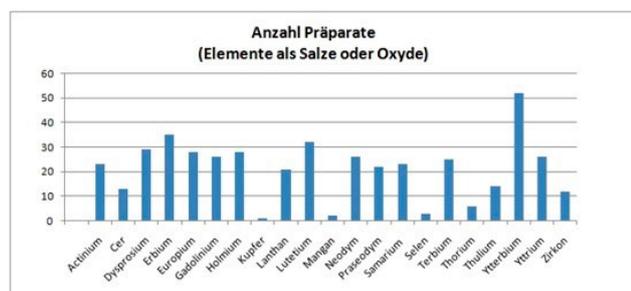


Figure 3. An overview of the compounds of the rare earth metals, which Carl Auer von Welsbach made available to other researchers. Number of compounds (element as a salt or oxide). Source¹⁵: Gerd Löffler, *Carl Auer von Welsbach und sein Beitrag zur frühen Radioaktivitätsforschung und Quantentheorie*, ISBN 978-3-200-04400-5, 2015, p. 126.

to assist these developments and their representatives, such as Ernest Rutherford and Niels Bohr for example. He strove to preserve science institutes with considerable donations where the need after the First World War was greatest, such as the Radium Institute of Vienna.⁶

In 1926, he published the results of his search for the rare earth element No. 61 (Atomic number 61).⁷ With the experimental methods at the time, there was nothing that could be detected. Von Welsbach was to be proved right. This gap in the periodic table of elements was able to be filled only in 1945 by producing 61 (promethium) using the nuclear reactor at Oak Ridge, USA. It is evident that a stable element (isotope) does not exist.⁸

Auer von Welsbach was distinguished in his lifetime with many honors. He was recognized not only in Aus-

tria, but also throughout Germany for his many achievements, such as, for example, with the “Siemens-Ring” (1921) which was unofficially designated as the German version of the Nobel Prize. He enjoyed an exceptionally high reputation with the Gesellschaft deutscher Chemiker (GDCh), who also hailed him and his accomplishments as a researcher and as a businessman on the 150th anniversary of his birthday in 2008.^{9,10} Auer von Welsbach was a member of various academies in Europe, e.g. from 1911¹¹ he was a full member of the Kaiserliche Akademie der Wissenschaften in Austria, and from 1913¹² – appointed by Max Planck – he was a corresponding member of the Preußische Akademie der Wissenschaften in Germany.

Carl Auer von Welsbach passed away in Welsbach castle, near Althofen, on 4th August 1929, and was buried in Vienna.

2. COLLABORATION WITH AND SUPPORT OF NIELS BOHR'S WORK

It is little known that Auer von Welsbach carried out pioneering work in the field of early quantum theory and made a considerable contribution as an experimental chemist to the slowly developing quantum theory of Max Planck from 1900, and then from 1910 to 1913 to that of Niels Bohr, culminating in an initial high point. He made numerous preparations of the rare earth elements for the physicists and chemists in Europe at that time in his laboratory in Carinthia in particular, in order to test the new theory in Copenhagen and Cambridge. The gratitude expressed by Niels Bohr to Auer von Welsbach in 1923 (see Fig. 4) was made at a time when George von Hevesy and Dirk Coster had discovered the long sought-after element hafnium in a zirconium mineral at the end of 1922, following previous extensive X-ray spectroscopic investigations, as well as the Welsbach preparations. This rare document underlines the connection between the Carinthian physicist and early quantum theory at that time.

2.1 Carl Auer von Welsbach's incandescent gas mantle and Max Planck's radiation formula. Forgotten details from the beginnings of quantum theory

Chemists and physicists, who do not or have not concerned themselves in particular with quantum physics, are mostly unaware why Carl Auer von Welsbach was so closely linked in the early phases of quantum theory with researchers in this field. Fig. 1 gives a short insight when names such as Bohr, Rutherford, Hevesy,

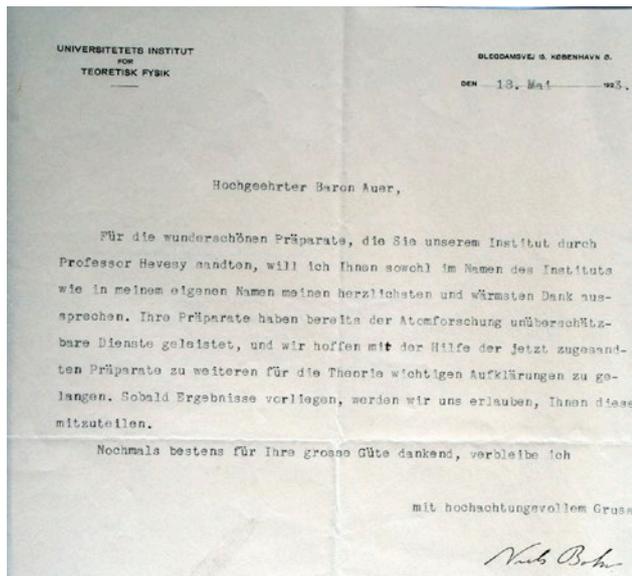


Figure 4. Niels Bohr to Carl Auer von Welsbach (05/13/1923). Source: Archives Auer-von-Welsbach Research Institute, File: Correspondence; Location: Auer-von-Welsbach Museum, Althofen (Carinthia).

Coster, Aston, and Siegbahn arise as recipients of von Welsbach preparations. Who remembers today that even the “father” of Japanese nuclear physics, Yoshio Nishina (1869-1951), investigated rare earth element preparations from Carl Auer von Welsbach by X-ray spectroscopy during his studies in Copenhagen with Niels Bohr from 1923 onwards?^{16,17}

The discoveries of the Planck radiation formula in 1900 and of the photoelectric effect (Einstein, 1905) can be considered turning points in our concept of the world of physics at that time. The rare earth elements and their final classification in the periodic system of the elements was a “real test” for the further development of nuclear and quantum physics at the beginning of the 20th century. If it had not been for the achievements of the chemists, who had specialized in the discovery and isolation of these “rare” metals – including in particular Carl Auer von Welsbach – quantum physics would not have been able to take the well-known dramatic developments in the first two decades of the last century. As one example for many other documents, this assessment arises from the letter from Niels Bohr to Carl Auer von Welsbach dated May 13, 1923 (see Fig. 4).

2.1.1 The interest of quantum physicists in the Auer light

The step taken by the natural sciences in the field of quantum physics was not very straightforward. It start-

ed in retrospect with the radiation laws of the physicist Gustav Kirchhoff governing the radiating properties of solid bodies. Together with Robert W. Bunsen, Kirchhoff established Heidelberg’s reputation as a research center, where the scientific career of Carl Auer von Welsbach also began. In 1860, Kirchhoff recognized that the ratio of the emissivity to the absorption capacity for all types of radiation, independent of the material properties of the body and for a certain wavelength, only depends on the temperature of the body. In 1875, Kirchhoff was appointed professor of theoretical physics in Berlin, which later became the center for quantum physics. Kirchhoff’s radiation law, with which theoretical physics introduced the term “black body,” as an ideal state, which a body approaches with increasing temperature,¹⁸ was the starting point for further considerations and experiments of physicists to describe the energy emitted by a body with a comprehensive law, which was finally presented on December 14, 1900 as the radiation formula at a convention of the German Physical Society by Max Planck. Kirchhoff himself sensed the fundamental significance of his findings and/or his radiation law and knew in which direction further research would have to be carried out in order to find a generally valid radiation law. In 1860, he wrote in this respect: “It is a task of great importance to find this function.¹⁹ Not until we have solved this problem, will we be able to reap all the fruits of our labors.”²⁰

The time span of about 40 years alone exemplifies the efforts, struggles and aberrations which had been experienced in order to come to a conclusive result. Starting with Kirchhoff, several physicists, such as Wilhelm Wien (1864-1928), Friedrich Paschen (1865), Otto Lummer (1860-1925) and Heinrich Rubens (1865-1922), were involved. Max Planck was also not immune to making errors until he decided “in an act of desperation”²¹ to introduce two natural constants: namely, the Boltzmann constant k ²² and the action quantum h , which had already been described in 1900, into the previously unsatisfactory drafts for a radiation formula and as such open the door to quantum physics. The main problem was to bring experiment and theory into agreement with one another in the infrared range. If, however, Kirchhoff had not introduced the “black body” and its physical properties (see above) in 1860, most certainly years, if not decades would have passed before this era of physics would have come up with the breakthrough insight created a completely new world view from 1900 onwards.

Producing a body which came closest to the “black body” postulated by Kirchhoff, was by no means trivial. There were many failed attempts in this respect. In par-

ticular, Wilhelm Wien and Otto Lummer attempted to find a solution at the Physical-Technical Imperial Institute in Berlin. The same applied from an experimental point of view to the development of a photometer, which could measure the wavelengths of radiating bodies with sufficient accuracy. Thermally insulated (heated) platinum tubing was finally used as a black body and as of 1889 a newly developed photometer was used, the so-called “Lummer-Brodhun cube”.²³ As a radiating body emits electromagnetic waves in the ultraviolet to the ultra-red range, it was also necessary to filter out the wavelengths from this spectrum with sufficient precision and intensity. In the development of this technology, the physicist Heinrich Rubens distinguished himself in particular with the residual radiation method. The aim was to clarify the discrepancies between theory and experiment in the ultra-red range. The method developed by Rubens was particularly suitable for this purpose. In the experiments, light sources and temperatures were necessary which could provide an ultra-red range with sufficient intensity. For this purpose, Rubens used the Auer light, i.e. the incandescent gas mantle.^{24,25} The Auer light is an excellent source of radiation in that only a small part of the emitted energy is in the visible range. Most of the energy is emitted in the ultraviolet and in particular in the infrared and the ultra-red range – which in this case was of great interest. Fig. 5 shows the principle of selective electromagnetic radiation for a certain wavelength starting from an emitter. Fig. 6 shows why the Auer incandescent mantle possesses suitable radiation capacity especially in the ultra-red range. The residual radiation method is based on the fact that some crystals have a selective reflectivity, such as e.g. rock salt, fluorspar and quartz. By positioning several plates of these crystals behind one another (multiple reflectivity), stray rays are suppressed and only electromagnetic waves of a certain wavelength are forwarded with a high degree of reflectivity (to a thermocouple). With an Auer

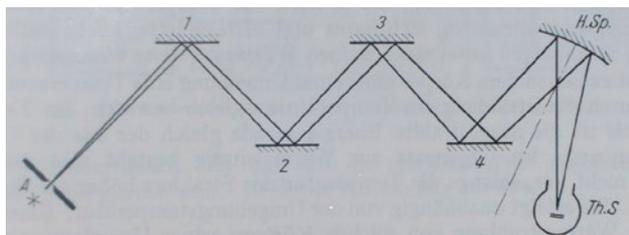


Figure 5. The residual radiation method according to Heinrich Rubens. A: Emitters, e.g. Auer incandescent mantle, H.SP: Concave mirror, Th.S: Thermal column, crystal plates 1, 2, 3, 4. Source: Gerthsen, Christian; Kneser, H. O.: “Physics”, Textbook, (Berlin 1969), 371.

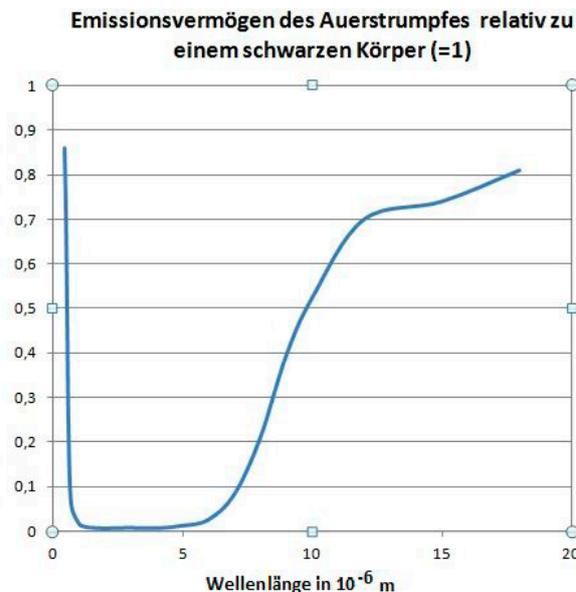


Figure 6. The emissivity of the Auer incandescent mantle relative to a black body (=1). Wellenlänge = wavelength 10^{-6} m. Source: Rubens, Heinrich: “The emission spectrum of the Auer incandescent mantle”, in: Proceedings of the German Physical Society 7 (1905), Table, 349.

light, one can thus create very precise radiation with a wavelength of $\lambda=43 \mu\text{m}$ with sufficient intensity. Optical grids were not suitable for this purpose.²⁶ In order to underscore the advantage of the Auer light using the residual radiation method, Heinrich Rubens is quoted as follows (extract): “As I explained earlier, one already obtains very pure residual rays after three-fold reflection of the radiation emitted by the Auer burner in a strength which amounts to 1.7 % of the entire radiation. A perfect black body at 1800 degrees abs. would result in less than 1 per mille of residual radiation under the same circumstances.”²⁷

The significance of the “radiation physicists” in Berlin, in particular Heinrich Rubens, and the experimental physics advanced by them cannot be praised highly enough for the acceptance of quantum physics, which gave rise to a completely new way of looking at processes on an atomic scale.

2.2 Quantum theory and magnetism of the rare earth elements

A short introduction to the subject of “magnetism”

It is general knowledge today that the earth possesses a magnetic field and that anyone can navigate themselves through the earth’s magnetic field with the help of a

compass using the direction given by the magnetic needle. In the same manner, it is general knowledge that a magnetic field can be created using simple means with a current flowing through a coil, whereby the coil likewise behaves as a magnetic needle in the earth's magnetic field if it is freely suspended. It is also known for objects of everyday life, e.g. in electronic products (televisions, computers), from food processors to cars, as well as in large manufacturing components, such as machines in all branches of industry, not least of all in power generating plants such as those utilizing atomic, thermal, water and wind energy, that all of the above use magnets and/or metals and metal alloys with magnetic properties.

There are four types of magnetism which can be differentiated: diamagnetism, paramagnetism, ferromagnetism and ferrimagnetism,²⁸ depending on the different physical properties of the metal compounds which occur in nature, or artificially manufactured metal oxides and metal alloys. For a long time, the causes of the different magnetic behavior of substances, i.e. the elements and their compounds, was not known. The technical applications were far ahead of an understanding of the actual physics.²⁹ Up until today, magnetism is a special field of solid state physics and the subject of further developments in quantum mechanics.

At this point in time, ferromagnetism, which is a characteristic of the substances iron, cobalt, nickel as well as the rare earth element gadolinium, and in which a magnetic force can also be determined without any external influence (without an additional external magnetic field), will not be further discussed here, as early quantum theory could not provide any conclusive explanation for this. The same applies for ferrimagnetism.³⁰ The situation is different with the diamagnetism of the elements, characterized by diamagnetic substances which are crowded out of an existing external magnetic field ($\mu < 0$; $\kappa < 1$).³¹ Paramagnetic substances are drawn into an existing magnetic field ($\mu > 0$; $\kappa > 1$). Experimentally, in the first case, there is attenuation of the existing magnetic field and in the second case it is intensified. Since the causes of this were not being pursued at that time (i.e. quantum physics was not a topic of interest), the differing behavior of diamagnetic and paramagnetic substances was described as "Lenz's principle",³² whereby it should be mentioned that both element types cannot be differentiated in their magnetic behavior if no external magnetic field is present. However, there is a further characteristic difference: diamagnetism is independent of temperature, whereas paramagnetism decreases with increasing temperature (Curie's Law). Further, it should be remembered that some paramagnetic substances (compounds) have a characteristic color.

Besides the elements of the iron group, the rare earth elements can also be included with those in the periodic system that are characterized by their special magnetic behavior (see above), with which Carl Auer von Welsbach had worked on so intensively and/or was decisively involved in their preparation in a pure state.

The understanding of magnetism only changed slowly and stepwise when the French physicist Paul Langevin (1872-1946) assumed from an atomic model in 1905 that an electron gas is a prerequisite for non-magnetic substances, whereby the quotient of the charge of the electrons to their mass is more than one thousand-fold greater than that of the compensating positive particles. Both particle types ensure external electrical neutrality. According to the classical theory, the electrons move in a circular motion (Larmor frequency) around an external magnetic field. These circular motions produce a magnetic moment. From this, the magnetic susceptibility for diamagnetism can be derived, as it still applies in principle today. It became apparent that the assumption of Langevin in this form was not tenable, as the properties of an electron gas residing around a positive charge had not been completely described. Such a model assumed by Langevin creates an opposite dipolar moment by the (spherically shaped) surface of the electron gas, so that the entire magnetic moment is zero. Niels Bohr referred to this in his dissertation in the year 1911.³³ Further steps in the development of nuclear and quantum physics were needed (Bohr-Rutherford, Bohr-Sommerfeld atomic model) as well as the step to quantum mechanics by Werner Heisenberg, Max Born, amongst others, in the mid 1920's. However, Langevin still deserves credit for his attempts and his conclusion that the magnetic behavior of the elements can be attributed in principle to that of the electrons.

2.3 The long road to a first quantum physical model for the magnetic behavior of the REE

Despite the progress that quantum theory had made, thanks to the famous theorists such as Bohr, Sommerfeld and Heisenberg, until the middle of the 1920's the abnormal Zeeman effect (splitting of the spectral lines in a strong magnetic field into more than 3 terms),³⁴ the Paschen-Back effect (multiple splitting of the spectral lines in a strong electrical field)³⁵ and the complicated spectra of the rare earth elements had not yet been disentangled. It was thus appropriate to understand the magnetic properties of these elements, starting with the previous findings about the numbers of occupied electrons according to the different main quantum numbers (= "electron shells") of the lighter elements and the empiri-

cally obtained magnetization numbers (or magnetic susceptibility).³⁶ The physicist Friedrich Hund (1886-1997) in Göttingen, a student of the theorist Max Born (1882-1970), was particularly specialized in this field. Hund published the results of his work in 1925 (see below).³⁷

It was a long road which had to be taken until this point in time was finally reached:

Bohr had already ascertained in his dissertation in 1910,³⁸ that the magnetic properties of the elements known then could not be explained using classical theory (with free electrons or those bound to atoms). According to classical theory (Langevin's theory of paramagnetism), it could be derived that all elements must have paramagnetic properties. In reality, however, the situation was and still is very different: most elements are diamagnetic.³⁹ The supporters of quantum theory were thus challenged again, and further development of quantum theory was urgently needed. This process concerned researchers, both opponents and advocates of quantum theory, for many years to come.

This discrepancy was addressed by Niels Bohr in several stages. In 1913, he went one step further by establishing a fundamental postulate. His solution to the further development of quantum theory was to assert the consistency of the angular momentum of an electron in an orbit, also in the presence of an external magnetic field. It can be shown using perturbation theory (in principle a mathematical process) that under this assumption one individual electron (of an atom) performs work against the external field,⁴⁰ i.e. the diamagnetic substances are crowded out of the (external) magnetic field. Physics only allows this explanation for the occurrence of diamagnetism, however, if the external magnetic field is vertical to the circular electron level. In other words, a theoretical model was only imaginable for one special case. Bohr's quantum theory had reached its limits.⁴¹ Progress in quantum physics was needed. This work was essentially carried out by Sommerfeld and subsequently by Max Born (1882-1970)⁴² and his students Werner Heisenberg, Pascal Jordan, Friedrich Hund (scientist for theoretical physics in Göttingen) as well as by the Austrian Wolfgang Pauli.^{43,44} From 1925 onwards, the step from quantum theory to quantum mechanics had been fulfilled. Besides Copenhagen, Göttingen became a center for theoretical physics.⁴⁵ The Stern-Gerlach experiment and that of Samuel Goudsmit and George Eugene Uhlenbeck were correctly interpreted in the course of this development. A difficult process from theory and experiment now led to the belief that a half-integral angular momentum ($= \frac{1}{2} h/2\pi$) and thus a magnetic moment had to be attributed to an electron. This was in no way to be taken as a matter of course, as up

until then electrons and protons were still considered to be the building blocks of the atomic nucleus (Thomson's atomic model).

2.4 Comparison of the magnetic properties of RRE: theory versus experiment

As already mentioned, Carl Auer von Welsbach discovered the rare earth element cassiopeium (called lutetium today) with the atomic number 71. In addition to other preparations, Auer von Welsbach also sent this preparation to the Institute for Radium Research in Vienna, where the head - Stefan Meyer - was working on the measurement of the magnetic properties of different elements, amongst other things.⁴⁶ On 2/29/1924, Meyer wrote to Auer von Welsbach:⁴⁷

*Institute for Radium Research" Vienna, 2/29/1924
IX., Boltzmannngasse 3.*

Dear Doctor!

As I already wrote to you, your pure $Cp_2(SO_4)_3 \times 8 H_2O$ is diamagnetic, which is of great interest. Likewise, HfO_2 is diamagnetic.⁴⁸

With kindest regards and greetings,

Yours faithfully,

Stefan Meyer

A few days later, on 3/6/1924, Carl Auer von Welsbach replied:

It will greatly please Bohr to hear that his prediction about the diamagnetism of Cp has been confirmed.⁴⁹

The letter from Auer von Welsbach to Bohr is one of the few direct pieces of evidence amongst the documents still remaining which shows that Auer von Welsbach was aware of the significance of the explanation of magnetism by quantum theory. Following this, Stefan Meyer published one year later the magnetization numbers of the other rare earth elements starting with atomic number 57 (lanthanum) up to atomic number 71 (cassiopeium = lutetium).

Meyer compared the measurements with the results of the famous Spanish physicist Blas Cabrera (1878-1945), who can also be included in this small circle of specialists in this area. The results concurred well, if one takes into account the state of measurement technology available at that time (see Fig. 7).

Remarks: Regarding the magnetization numbers (Weiss's magnetons)⁵¹ of the rare earth elements (lanthanum La to cassiopeium Cp), see the following footnotes.^{52,53}

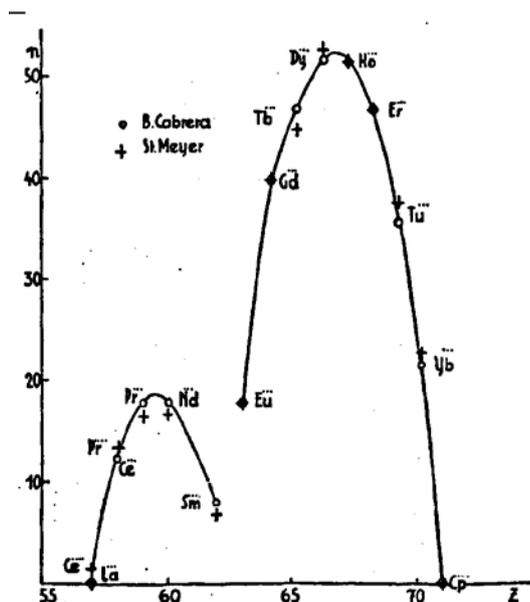


Fig. 7 The magnetization numbers (Weiss's magnetons) of the rare earth elements (lanthanum ^{57}La to cassiopeium ^{71}Cp). n : Weiss's magnetons; Z = atomic number (= number of protons). Source⁵⁰: Stefan Meyer, Magnetization numbers of the rare earths, *Physikalische Zeitschrift*, 1925, 26, p. 53.

Stefan Meyer (1925) wrote in the introduction:

Since the discovery of the unusual magnetic properties (1899¹) of the rare earths, I have always had the opportunity to measure their magnetization numbers thanks to the kindness of C. Auer-Welsbach and have repeatedly reported about this.²

1) Vienna. Ber. (IIa), 108, 861, 1899 2) Vienna. Ber. (IIa), 109, 403, 1900; 110, 541, 1901; 111, 38, 1902; 117, 995, 1908)

Meyer continues: "At the beginning of 1924, I informed the interested parties, especially the Bohr Institute in Copenhagen, that both cassiopeium (71) as well as hafnium (72) are diamagnetic, and that in the first instance this information referred to Cp preparations from 1915 and 1924."⁵⁴ Niels Bohr had already recognized the significance of the Welsbach preparations in 1923, when he classified them as "inestimable" for atomic research (see the letter from Bohr to Auer von Welsbach dated 05/13/1923, Fig. 4).

Experimentally obtained data on the magnetic behavior of the rare earth metals were now available and could be considered essentially verified according to the state of measurement technology at that time. However, the theoretical explanation was still missing.

In 1925, the physicist Friedrich Hund succeeded in essentially explaining theoretically the experimental

data which had been collected up until then about the magnetic behavior of the above-mentioned metals on the basis of the work of the afore-mentioned physicists in Göttingen, the assumptions of Niels Bohr about the successive occupation of the subjacent electron shells (today described as the 4f-electrons),⁵⁵ furthermore taking the Pauli principle and especially the preliminary work of Heisenberg into consideration.^{56,57} Hund described his theoretical model in an abridged form, as follows: "Shortly thereafter, he [Hund] was able to explain the magnetic behavior of the rare earths by giving the basic state of their trivalent ions using the rule that from the possible multiplets with the deepest energetic configuration, the multiplets with the highest multiplicity lie deepest and that which is the deepest is the one with the highest angular momentum."⁵⁸

Hund established the Bohr magneton numbers (for the trivalent ions of the rare earth elements lanthanum and subsequently) and then converted these into Weiss's magneton numbers.⁵⁹ In this way, he could compare his theoretically established values with the data determined experimentally by Blas Cabrera and Stefan Meyer. The comparison with the data from Stefan Meyer is shown in Fig. 8.⁶⁰

The level of agreement between theory and experimental results was astoundingly good considering the experimental techniques used by Stefan Meyer and the state of quantum theory at that time. In particular, the occurrence of two maxima by Hund could be comprehended using one of the theories based on quantum physics. Both lanthanum and lutetium ions are clearly diamagnetic.

There is, however, a noticeable deviation between theory and experiment for the europium ion. Stefan Meyer explained that the europium preparation used must have still contained 5-6 % gadolinium, in other words it was slightly impure. He states that for this reason, Weiss's magneton number would have to be 15.5 instead of 18.5.⁶¹ According to this, the difference between theory and experiment would then be reduced, although for the europium ion it would still be significant.⁶² The comments from Meyer, however, also show that through the advances made in measurement techniques the investigations of magnetic properties were suitable to identify the rare earth elements in substances and/or to establish their degree of impurity.

The actual objective to disentangle the spectral lines of the rare earth elements came one step closer by an understanding of their magnetic properties, namely by quantum theoretical specifications for the permissible energy states of an atom and/or these elements. In

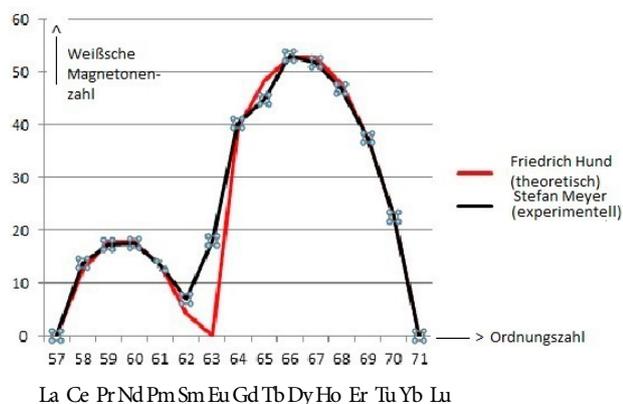


Fig. 8 Magnetic properties of the rare earth elements (trivalent ions of ⁵⁷lanthanum to ⁷¹lutetium). Source⁶³: Friedrich Hund, Atomtheoretische Deutung des Magnetismus der Seltenen Erden (= Theoretical atomic signification of magnetism of the rare earths), *Zeitschrift für Physik*, **1925**, 33, Table 1, p. 857.

this respect, the quality and purity of the preparations of these elements (mostly sulfates) which were also supplied by Carl Auer von Welsbach played an important role. However, it must be brought to mind here that the arc and spark spectra, e.g. of dysprosium and yttrium compounds exhibited more than 3,000 and/or more than 2,000 lines, respectively.⁶⁴ However, the spectra of the rare earth elements were not fully explained during the lifetime of Carl Auer von Welsbach. Further advances were linked to continued developments in quantum mechanics and achieved after his death in the 1930's.⁶⁵

Today, rare earth elements are used in numerous industrial products (in the form of complex metal compounds) due to their magnetic properties, in particular in the electronics industry. This is referred to as an inherent “magnetic technology”. This rapid development was unimaginable at that time up to the end of the 1920's. However, it shows the true significance of basic research work which was carried out almost 100 years ago.

3. THE DETERMINATION OF THE ATOMIC WEIGHTS OF YTTERBIUM AND LUTETIUM

The two elements ytterbium and lutetium conclude the lanthanide series. These findings and the properties of these two elements still have an important significance for chemists and physicists today. The discovery and the following chemical-physical investigations commencing with ytterbium by Jean Charles de Marignac (1817-1894)⁶⁶ in a mineral obtained from the area around the Swedish town of Ytterby near Stockholm (1878) and finally the separation of this element by Auer von Welsbach commencing 1905-1914 – almost at the same time as the French chemist Georges Urbain – into ytterbium as it is known today (called aldebaranium at that time) and lutetium (called cassiopeium at that time) has given today's table of the elements its definitive structure through modern chemistry and physics.

Auer von Welsbach had already worked on ytterbium relatively early on, as can be seen from the records of his most important employee, Ludwig Haitinger (1860-1945).⁶⁷ Here it is briefly described how he separated the original ytterbium from the elements scandium and erbium contained in the starting mineral. Auer was also aware of the weak basicity of ytterbium before 1893. He also knew that the oxalate of this supposed element (a white powder) was almost insoluble in water and dilute hydrochloric acid. Besides the knowledge of the spectrum, these findings served him later on in his further work to determine the atomic weight of the actual elements, namely ytterbium and lutetium. The procedure used for the separation of ytterbium (old) into ytterbium (aldebaranium) and lutetium (cassiopeium), starting from half a ton of crude ytteroxalate in 1906⁶⁸ is described in detail in the *Sitzungsberichte der Kaiserliche Akademie der Wissenschaften*.

The “birth” of the new element, in this case the discovery of the element lutetium, can best be described by the discoverer in his own words, and therefore his description of this moment and/or the time at the beginning of 1905 should be reproduced verbatim, similarly his description of the first atomic weight determinations in 1906 should be quoted. These were repeated in

Table 1. (to Fig. 8).

Atomic number	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
Mg. number Cabrera	0.0	12.5	17.8	17.8	13.4	4.2	0.0	39.4	48.3	52.8	52.8	47.7	37.6	22.5	0.0
Mg. number Meyer	0.0	13.8	17.3	17.5	13.4	7.0	18.0	40.2	44.8	53.0	51.9	46.7	37.5	22.5	0.0

Mg. number = Weiss's magneton number

the years to follow. Several records from 1912 about the determination of the atomic weights of ytterbium (aldebaranium) and lutetium can be found in handwritten documents, which are archived in the Carl Auer von Welsbach Museum (see Fig. 9, Fig. 10 and the literature references below).

The earliest announcement of this discovery of the composition of the old ytterbium from two elements results from the notification of these findings by Auer von Welsbach with the Imperial Academy of Sciences in Vienna in 1905. However, this did not yet include the atomic weights and spectra of the two elements. For this reason, however, we know that mainly the spectral findings at this point in time supported these results.⁶⁹ In 1906 and 1907, he then described how he proceeded exactly and his results (spectra, atomic weights of the newly discovered elements, in this respect see also below).

In 1906, readers were informed in a separate treatise via the journal 'Justus Liebig's Annalen der Chemie' about the exciting situation in his laboratory at the time of the discovery.⁷⁰ Here he wrote:

During the investigation of ytterbium ammonium oxalate, I noticed some strange phenomena which suggested that ytterbium is not a uniform body. I first compared the different fractions (remark of the author: fractionated crystallization) amongst themselves and also using pure ytterbium from earlier presentations, but could not find any indications for the correctness of my assumption despite careful comparison. Nevertheless, I continued the fractionation. After a longer period of time, I obtained a preparation which demonstrated quite distinct changes in intensity, which in the case of weak arcing showed up by all means in some of the characteristic lines in the red of ytterbium in comparison to other ytterbium preparations. When this separation process was continued, these changes became even more striking. Individual lines in the red started to become paler, others became more pronounced with all the more radiance. Now the change in the intensity of the lines became noticeable also in the remaining parts of the spectrum.

When I then compared the ytterbium fractions which were farthest apart later on, after protracted continuation of the separation process, the distinct differences of the two spectra came to light. With the continuation of the separation process, I realized that the purest preparations did not have any common lines any longer even with strong arcing in the optical part of the spectrum.

I now photographed the spectra which had been produced with the help of a concave grid under fully identical experimental conditions in the range $\lambda = 4500\text{-}2600$ (Å).⁷¹ The glass-clear, very high-contrasting negatives gave, when compared directly, an extremely interesting picture. Almost all lines were very precise upon moderate enlargement ...

Following a short description of the spectra, he continued

Thanks to these explanations which were most certainly needed, the exact scientific proof of the successful separation of ytterbium into two bodies had been provided.

Auer von Welsbach then established that he "had already informed anyone who had asked in the year 1906", and that he had informed them about the approximate wavelengths and the approximate atomic weights. He had determined the atomic weights for Ad= 172.52 g/mol (ytterbium) and Cp= 174.28 g/mol (lutetium).

The separation method he used was fractional crystallization (Appendix). This method was also suitable, amongst other things, as the rare earth elements all crystallize isomorphically. His particular contribution to the further development was that he used the different solubility of ammonium binoxalate of the rare earths in ammonium oxalate (intentionally mixed with some ammonia so that no turbidity occurred)⁷² during fractionation and was thus able to follow in many hundreds of steps the slow separation by continuous analysis of his (arc) spectra,⁷³ until no change in the spectra of the elements ytterbium (new) and lutetium could be observed.

In the years that followed up until 1914, very exact spectra and atomic weights which had been determined by further experiments were published. The publications not only appeared in the communications of the Imperial Academy of Sciences but also partly in other scientific journals.⁷⁴

The rare earth elements which are discussed here were certainly available to him in 1903 through the basic maceration of the raw material monazite (using sodium hydroxide)⁷⁵ as oxides and/or as salts, even though he did not expressly mention this. Monazite was the starting material for the extraction of thorium, which was on the one hand the main constituent of the incandescent mantles of his gas lights which were sold and used on a worldwide basis. During their production, different mixtures of the rare earths, amongst other things, accumulated copiously as a "waste product". The French chemist Marginaç (who had previously also discovered gadolinium and was very well known) succeeded in extracting the old ytterbium from these mixtures in 1878, as already mentioned above.

In 1907, Auer von Welsbach published a comprehensive paper and determined thereby the atomic weights of ytterbium (aldebaranium) and lutetium (cassiopeium) as 172.90 and 174.23 on the basis O=16 according to the Bunsen method. In the same publication, he also published the corresponding spectra.⁷⁶

In 1912, Auer von Welsbach made a further attempt to determine the atomic weight of the elements ytterbium (new) and lutetium, which he had extracted from the old ytterbium in 1905, apparently in order to improve the accuracy of his procedure and thus the result. The route that he took is to be looked at more closely here:

After conclusion of the fractional crystallization, which extended as far as the 320th series, he still had various ytterbium and lutetium preparations from 1903 in the form of solid substances (oxalates, oxides) and (not further fractionated) lyes. In a renewed spectroscopic examination, he noticed that some spectra of these preparations still showed slight traces of calcium (Ca), sodium (Na), zinc (Zn) manganese (Mn) and thorium (Th) as well as several traces of silicic acids (SiO₂). For the determination of the atomic weights, Auer von Welsbach concentrated on the Cp (lutetium) preparations which were still slightly impure with silicic acid amounting to 18.2 grams and which originated from the end fractions from the 147th – 319th series. The elimination of K and Na succeeded by the skillful addition of nitric acid and hydrogen sulfide. Finally, a few steps later, a Cp-nitrate solution was produced which was precipitated with ammonia. A part of the hydrate was dissolved in nitric acid, the other part was smelted. The oxide Cp₂O₃ (Lu₂O₃) was obtained. The K and Na traces had been eliminated in this Cp preparation.⁷⁷

In order to remove the traces of silicic acid, the purification process had to be extended and continued with other substances. The Cp oxide obtained in the first step was dissolved again in nitric acid. The solution was concentrated and heated until the evolution of nitric oxide. The smelt was partially dissolved in water. In addition to the turbidity of the water, a flaky precipitate occurred in a small amount, which did not dissolve after the renewed addition of nitric acid or water, and no turbidity of the liquids could be observed. It could be proven that the SiO₂ traces (and also thorium traces) had been at least partly removed with the precipitate. Finally, Auer von Welsbach used oxalic acid to which nitric acid had been added twice in a row while heating the precipitate to incandescence, with precipitation of the respective mother liquor. By “vigorous” heating to incandescence of the last oxalate precipitate in a platinum crucible, he obtained Cp oxide as a white powder. “Contrary to expectations”, however, the silicic acid had not entirely disappeared. At this time, Auer von Welsbach had a platinum crucible, which he had used for “more than 30 years”, but the impurities in the platinum itself could not be completely excluded. By weigh-

ing the crucible before and after heating to incandescence, he could exclude such an effect. The accuracy of his weighing activities must have amounted to approx. \pm 0.001 grams.

The Cp oxides were now dissolved in nitric acid. This clear solution was mixed with sulfuric acid in a slight overage and slowly concentrated. “Crystal clear” and “nice-looking” sulfate crystals were formed. These were then dissolved in a little water so that a completely clear solution was obtained. In the last step, oxalic acid “free of ash residues” was added to this solution. The Cp oxalate was precipitated, was washed and finally heated to incandescence. The result was Cp₂O₃ (Lu₂O₃) as an oxide which had been purified several times from the traces of other elements.

Auer von Welsbach had now produced lutetium (Cp) in different compound forms starting from a defined amount, namely as a defined amount of hydroxide, sulfate and as oxide. The last two compounds alone, whose exact amounts he determined gravimetrically, would have sufficed in order to determine the atomic weight of lutetium (Lu) by a simple calculation. In a similar manner, he continued experimentally with the original ytterbium (new) that he had produced in 1905. In 1912/1913 he repeated these experiments a total of three times with essentially the same results. The details of the atomic weight refer to the atomic weight for oxygen O = 16. On average, these experiments resulted in 173.00 for ytterbium and 175.00 for lutetium.⁷⁸

The handwritten laboratory records used for the determination of the Lu atomic weight from the year 1912 are still mostly available and are archived in the Auer von Welsbach Museum. These are to be considered according to the sources given as an obvious preliminary result of the final results, which were published in June 1913 in the monthly edition of the *Chemie Mitteilungen* (Chemical Communications)⁷⁹. It is apparent here, amongst other things, that Auer von Welsbach had gradually come closer to the final result through his corrections, using as an example the element lutetium, (cassiopeium = Cp), (see Fig. 9 and Fig. 10)

Compared to the current values used today for Yb = 173.045(10), Lu = 174.9668, this is to be considered an outstanding result considering the technology available to chemists at that time in 1910.⁸⁰

One of the highlights of the exhibits in the Auer von Welsbach Museum is that there is still a sealed test tube with a lutetium preparation (Cp₂O₃) originally produced by Auer (see Fig. 11)

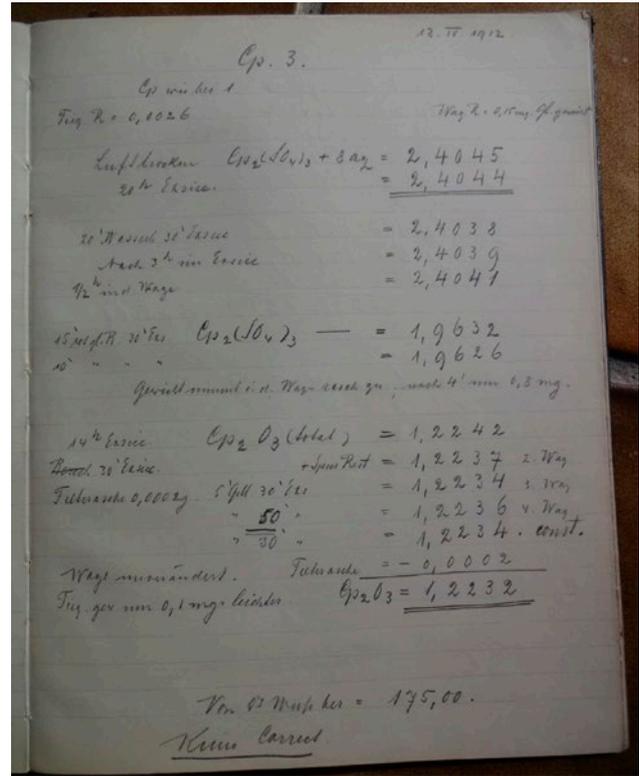
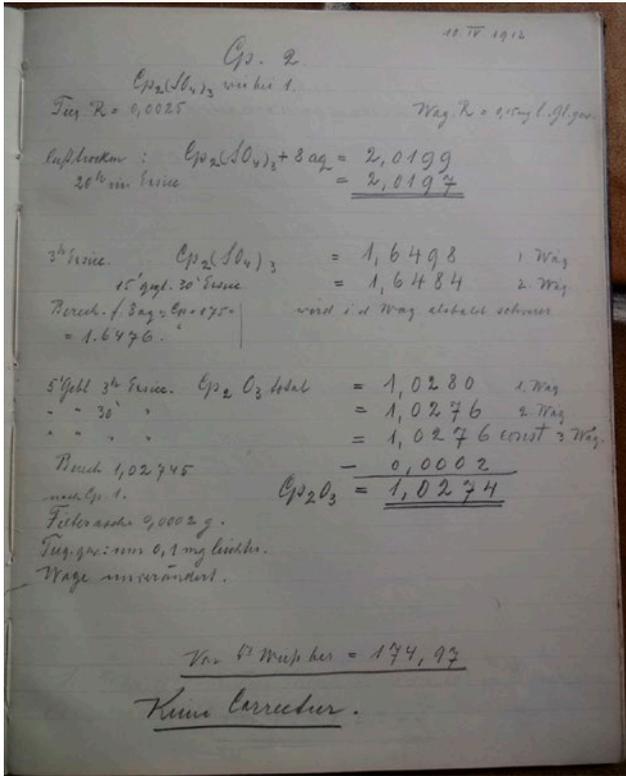


Figure 9. Lutetium (Cp), atomic weight determination, second to last step 1912.

Figure 10. Lutetium (Cp), atomic weight determination 1912 (last step).



Figure 11. Test tube with a lutetium preparation (Cp₂O₃) originally produced by Carl Auer von Welsbach.

APPENDIX: FRACTIONAL CRYSTALLIZATION

The principal procedure used in the separation of substance mixtures by "fractional crystallization": A method to separate the rare earth elements and mixtures of radioactive substances.

Multicomponent mixtures, dissolved at an increased temperature until saturation, e.g. in distilled

water, nitric acid or in other solvents, can be separated into individual components, i.e. for example, into oxides or salts of the individual elements of the mixtures of these substances, in which this heated solution is cooled down (e.g. by allowing it to stand). As the components of this mixture of substances as a rule possess differences in solubility, depending on the temperature, these element compounds crystallize at different points in time. If the crystallization process is interrupted at the right time and the alkaline solution (mother liquor) is removed, one obtains the first fraction (top fraction) of an element - e.g. as a salt - which has become more concentrated and is relatively free of the other components of the original mixture. This precipitate is then re-dissolved and the procedure is repeated. The respective mother liquors are evaporated again and then cooled down as in the first step. Through the skillful combination of crystallization, dissolution and evaporation, the most hardly soluble element becomes increasingly concentrated and as such separates from the original mixture of substances. The sequence of this procedure can be seen schematically in Fig. 12.

The greater the number of crystallization steps, the purer the presentation of the individual components will

- Research Institute, File: Correspondence; Location: Auer-von-Welsbach Museum, Althofen (Carinthia).
17. Yoshio, Nishina, On the L-absorption Spectra of the Elements from Sn(50) to W(74) and their Relation to the Atomic Constitution, *Philosophical Magazine and Journal*, **1925**, 49, 522 f.
 18. In practice, hollow spheres and cylinders from fire-proof materials with circular openings of a few millimeters were used. These objects were kept at a constant temperature. The (electromagnetic) radiation coming from the opening was then analyzed in relation to the temperature. One analytic method (German: Reststrahlenmethode “residual rays method”) is shown in schematic form in Fig. 5.
 19. The increasing industrialization of the economy was another driving force to gain a clear understanding of the processes of heat and light generation. For example, in 1860, there was no safe, i.e. continuously glowing, illuminant yet. The gas mantle that was developed by Carl Auer von Welsbach was not patented until 1885 and was only introduced on the market in series production as a safe light source around 1891. Source: See online <http://www.althofen.at/welsbach.htm> (03/04/2019).
 20. Wilfried, Hauser, *Strahlungsphysiker ebnen in Berlin den Weg zur Quantentheorie – Vienna, Paschen, Lummer, Rubens*, in: Wilhelm Treue; Gerhard Hildebrandt, *Berlinische Lebensbilder “Naturwissenschaftler”*, Berlin, **1987**, p. 92.
 21. Armin Hermann, *Max Planck. Vorträge und Erinnerungen*, see online: <http://www.zeit.de/1984/08/vortraege-und-erinnerungen> (**24/03/2019**).
 22. Max Planck finally decided, after some hesitation, to build on the work of Austrian physicist Ludwig Boltzmann in the area of thermodynamics for the development of his radiation formula. Boltzmann was seen to be a proponent of “atomism.”
 23. See online <https://physicsmuseum.uq.edu.au/lummer-brodhun-photometer> (**23.04.2014**)
 24. Dieter Meschede, *Gerthsen Physik*, Berlin/Heidelberg, **2006**, p. 588.
 25. Heinrich Rubens; Friedrich Kurlbaum, Anwendung der Methode der Reststrahlen zur Prüfung des Strahlungsgesetzes, *Annalen der Physik*, **1901**, 4, 649-666.
 26. Meschede, **2006**, p. 588.
 27. Heinrich Rubens, Über das Emissionsverhalten des Auerbrenners”, *Annalen der Physik* **1905**, 14, 734.
 28. Horst, Stöcker, *Taschenbuch der Physik*, Frankfurt/Main, **2010**, pp. 441-447.
 29. Carl Auer von Welsbach built his first hydroelectric power plant in 1898 for generating electricity for the production of osmium lamps and the production of rare earth elements in his factory in Althofen-Treibach. For his construction of power plants in Carinthia, see Roland Adunka, Carl Auer von Welsbach. Entdecker – Erfinder – Firmengründer, **2015**, 64-67.
 30. Ferrimagnetism is the characteristic of crystals that have partial grids with opposing magnetization. They only compensate each other in part and are therefore only slightly ferromagnetic. Source: Horst Stöcker, *Taschenbuch der Physik*, **2010**, pp. 956 f.
 31. $\mu = \kappa - 1$ applies, whereby μ indicates magnetic susceptibility and κ indicates magnetic permeability – which is the relationship of magnetic flux density B (in the material) to the outer magnetic field strength H. The magnetic susceptibility μ is identical or rather proportional to the rate of magnetization used by many researchers.
 32. Fachredaktion für Naturwissenschaften und Technik des Bibliografischen Instituts (Ed.), *Meyers Physik-Lexikon*, **1973**, p. 525.
 33. Dieter Wagner, Das Rätsel des Magnetismus, *Physik Journal*, **2005**, 4/3, 23 f.
 34. The normal Zeeman Effect (division of a spectral line into 3 sections) has already been explained by means of the classical electrodynamics and early quantum theory, as the spin of electrons was not known at this time.
 35. Both the physicists Friedrich Paschen (1865-197) and Ernst Back (1851-1959) discovered the Paschen-Back effect in 1921.
 36. $M = \chi \times H_0 : \rightarrow \chi = M/H_0$ magnetic susceptibility, M= magnetic moment, H_0 = outer magnetic field
 37. Friedrich, Hund, Atomtheoretische Deutung des Magnetismus der seltenen Erden, *Zeitschrift für Physik* (**1925**), 33, 855-859.
 38. Niels Bohr, *Studies on the electron theory of metals*, (Diss.), **1911**, p. 395.
 39. Ulrich, Hoyer, *Die Geschichte der Bohr’schen Atomtheorie*”, **1974**, p.150.
 40. Ibid., p. 153.
 41. Ibid., p. 160.
 42. Max Born, *Mein Leben*, **1975**, pp. 288-304.
 43. Wolfgang Pauli, Über den Einfluß der Geschwindigkeitsabhängigkeit der Elektronenmasse auf den Zeeman-Effekt, *Zeitschrift für Physik*, **1925**, 31, 373-385.
 44. Wolfgang Pauli, Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren, *Zeitschrift für Physik*, **1925**, 31, 765-783.
 45. Werner Heisenberg, Über quantentheoretische Umdeutungen kinematischer und mechanischer Beziehungen, *Zeitschrift für Physik*, **1925**, 33, pp. 879-893 and Max Born; Pascal Jordan, Zur Quan-

- tenmechanik, *Zeitschrift für Physik*, **1925**, 34, 858-888. Max Born; Werner Heisenberg; Pascal Jordan, Zur Quantenmechanik. II., *Zeitschrift für Physik*, **1926**, 35, pp. 557-615. In 1933 Heisenberg, along with Erwin Schrödinger and Paul Dirac, received the Nobel Prize for Physics for the formulation and presentation of the “new” quantum mechanics.
46. The physicists Stefan Meyer and Egon v. Schweidler in Vienna were experimentally perfectly equipped for this. In 1899/1900, at around the same time as the French H. Becquerel and German chemist F. Giesel, they proved through the deflection of magnetic and electric fields, that radium rays (β -rays) behaved as though they were negatively charged (electrons). Through an attempt at deflecting the polynomial rays it was determined that they were twofold positively charge helium atoms, which were identified as α - rays.
 47. Letter from Stefan Meyer to Carl Auer von Welsbach dated **02/29/1924**, Archives Auer-von-Welsbach Research Institute, File: Correspondence; Location: Auer-von-Welsbach Museum, Althofen (Carinthia).
 48. Cp is the abbreviation for the rare earth element casiopeium, called lutetium (Lu) today. Hf is the abbreviation for the element hafnium, which will be dealt with later.
 49. Letter from Carl Auer von Welsbach to Stefan Meyer dated **03/06/1924**, Archives Auer-von-Welsbach Research Institute, File: Correspondence; Location: Auer-von-Welsbach Museum, Althofen (Carinthia).
 50. Stefan Meyer, Magnetization numbers of the rare earths, *Physikalische Zeitschrift*, **1925**, 26, p. 53.
 51. The French physicist Pierre-Ernest Weiß (1865-1940) had studied metals and their compounds (e.g. iron, cobalt, nickel, magnetite) intensively even before the beginnings of Bohr’s quantum theory. See: “Über die rationalen Verhältnisse der magnetischen Momente der Moleküle und das Magneton”, in: *Physikalische Zeitschrift* 12 (1911), 935-952. They demonstrated a strong magnetic attraction without an outer field, thus were ferromagnetic. His investigations led to the conclusion that in such bodies, small microscopic dipoles must exist, which are also equally aligned in very small areas (max. 1mm diameter). These areas are called Weiß’sche Bereiche (magnetic domains). Through external influences, for example through a magnetic field, a multitude of these dipoles can be aligned in different areas. In doing so we get to magnets used in electronics, dynamos and other propulsion machinery. The correct explanation for this characteristic was provided not till modern quantum mechanics. Rare earth elements play an important role in producing magnets of this kind. Above the so-called ferromagnetic Curie-temperature, which is a material constant, the magnets slowly lose their characteristics and become paramagnetic. Source:
 52. Horst Stöcker, **2010**, p.445, p. 661.
 53. Stefan Meyer measured the anhydrous sulfates of rare earth elements. In this compound the elements were trivalent, which is the preferred valency. Source: see Holleman; Wiberg, *Lehrbuch der anorganischen Chemie*, **2007**, p. 1939. (Nomenclature by Stefan Meyer e.g. for the lanthanum cation: La). The sulfate anions provide no contribution to magnetization. The cations La and Cp are diamagnetic. According to Stefan Meyer the remaining cations are paramagnetic. For the element with the atomic number 61 (promethium) no rate of magnetism could be determined, because this element in its extreme rarity was impossible to determine. See also Carl Auer von Welsbach, Über einige Versuche zur Auffindung des Elements Nr. 61, *Chemiker-Zeitung*, **1926**, 118, p. 990. Here magnetic susceptibility means the specification of magnetization (on the ordinate in Fig. 7), the value of the so-called Weiß magnetons. Stefan Meyer chose this measuring unit in order to be able to compare his results with those of Cabrera.
 54. Blas Cabrera had also received rare earth compounds from Carl Auer von Welsbach via the institute for radium research in Vienna. This emerges from a letter from Stefan Meyer to Auer von Welsbach from 1924. Cabrera paid the institute 300 pesetas for this compound. Stefan Meyer thanked Auer von Welsbach. Source: Letter from Stefan Meyer to Auer von Welsbach dated **07/25/1924**. Archives Auer-von-Welsbach Research Institute, File: Correspondence; Location: Auer-von-Welsbach Museum, Althofen (Carinthia).
 55. Stefan Meyer, Magnetisierungszahlen der seltenen Erden, *Physikalische Zeitschrift*, **1925**, 26, 51 f.
 56. The head of the radium institute in Vienna, Stefan Meyer, who had been working mostly with preparations from Carl Auer von Welsbach during his research on magnetic behaviors of rare earth elements since 1899, pointed out that he had already in 1915 expressed the assumption that the magnetism of these elements could not be explained solely by the valence electrons (in the outer shell of the atom) but that electrons in the inner shells had to play a role. Source: Meyer, **1925**, 52.
 57. Werner, Heisenberg, Zur Quantentheorie der Multiplettstruktur und der anormalen Zeemanefekte, *Zeitschrift für Physik*, **1925**, 32, 841-860.
 58. Friedrich Hund, Atomtheoretische Deutung des Magnetismus der seltenen Erden, *Zeitschrift für Physik* 33 **1925**, 33, 855-859.

59. Friedrich Hund, *Geschichte der Quantentheorie*, **1975**, p.117. These selection rules became referred to as Hund's rules. They are explained as follows: When adhering to the Pauli principal in an atom L equals the sum of all orbital angular momentum, S equals the sum of all electron spins and $J=L+S$, it can be concluded that:
1. the basic state takes the maximum possible total spin $S \rightarrow S_{max}$, d. h., it possesses the maximum possible total symmetry
 2. the basic state also possesses, along with rule 1, the maximal possible orbital angular momentum, $L \rightarrow L_{max}$
 3. The total angular momentum J takes on the lowest horizontal state of worth:
 $J = IL - SI$ with less than half full shell
 $J = IL + SI$ with more than half full shell
 Cf. Dieter Meschede, *Gerthsen Physik*, **2005**, p, 853. These rules can be deduced from quantum mechanics. A basic principle is that every atom takes the lowest possible energy state. Rule number three makes it also plausible, that in Fig. 7 two characteristic curved lines occur.
60. Hund, **1925**, p. 857 f.
61. Ibid., p, 858.
62. Meyer, **1925**, p.478 f.
63. In this respect, it should be noted that in 1925 it was not yet usual to determine measuring errors in an experiment in either systematic and statistical errors and publish them accordingly. In the present case besides contamination, other systematic errors could have had occurred.
64. Friedrich Hund, Atomtheoretische Deutung des Magnetismus der Seltenen Erden (= Theoretical atomic significance of magnetism of the rare earths), *Zeitschrift für Physik*, **1925**, 33, Table 1, p. 857.
65. Roughly cited from Georg Hevesy, *Die seltenen Erden vom Standpunkte des Atombaues*, **1927**, p. 80 f.
66. Vleck assessed the presented explanation for the magnetic behavior of the rare earth elements by Friedrich Hund in 1925 (s. o.) in his Nobel Prize speech in 1977 as follows "In 1925 Hund (...) wrote a paper on the magnetic susceptibilities of rare earth compounds which was the crowning achievement of empiricism of the old quantum theory." See: J.H. Vleck, Quantum Mechanics. The Key to understand Magnetism, *Science*, **1978**, 201,4341, p.115.
67. Ludwig Haitinger, *Handbuch der Anorganischen Chemie*, **1893**, III, p. 53.
68. Ibid.
69. Auer von Welsbach, Carl, Über die Elemente der Yttergruppe, (erster Teil), *Proceedings of the Imperial Academy of Sciences, Sect. II b*, **1906**, 115, pp. 737-747.
70. Carl Auer von Welsbach, *Vorläufiger Bericht über die Zerlegung des Ytterbiums*, Gazette of the Imperial Academy of Sciences, **1905**, 42, p. 122.
71. Commemorative publication *Adolf Lieben and Liebig's Annalen der Chemi*, **1906**, 351, pp. 464-465., also cited in: Carl Auer von Welsbach, Zur Zerlegung des Ytterbiums, *Chemie-Heft*, **1909**, 9, p. 49.
72. Auer von Welsbach does not give a unit of measurement. Since his spectral analytical research was conducted in the visible area, only $\text{Å} = 10^{-10} \text{ m}$ is a possible unit of measurement.
73. Carl Auer von Welsbach, Über die Elemente der Yttergruppe, (1. Teil), *Proceedings of the Imperial Academy of Sciences, Sect. II b*, **1906**, 115, pp. 737-747.
74. Auer von Welsbach published the spectroscopical technology that was further developed by him in 1922: Carl Auer von Welsbach, Spektroskopische Methoden der analytischen Chemie, *Proceedings of the Imperial Academy of Sciences, Sect. II b*, **1922**, 131, pp. 387-408.
75. Carl Auer von Welsbach, Über die Elemente der Yttergruppe, (1. Teil), *Proceedings of the Imperial Academy of Sciences, Vol. CXV, Sect. II b*, **1906**, 115, pp. 737-447. Carl Auer von Welsbach, Die Zerlegung des Ytterbiums in seine Elemente, *Proceedings of the Imperial Academy of Sciences, Vol. CXVI, Sect. II b*, **1907**, pp. 1425-1469. Carl Auer von Welsbach, Zur Zerlegung des Ytterbiums, *Proceedings of the Imperial Academy of Sciences, Sect. II b, Supplement*, **1909**, 118, pp. 307-312. Carl Auer von Welsbach, *Zeitschrift für anorganische Chemie*, **1911**, 86, pp. 58-70.
76. See online: https://commons.wikimedia.org/wiki/File:Monazit_Behandlung_basischer_Aufschluss.svg (02/04/2019)
77. Carl Auer von Welsbach, Die Zerlegung des Ytterbiums in seine Elemente, *Proceedings of the Imperial Academy of Sciences, Sect. II b*, **1907**, 116, pp. 1425-1469.
78. Auer von Welsbach was always in close contact with experts F. Exner, J.M. Eder and E. Valenta in Vienna, making his preparations available to them for further spectral analytical research. See e.g. J.M. Eder; E. Valenta, *Wellenlängenmessungen im sichtbaren Bereich der Bogenspektren der von Auer von Welsbach entdeckten Elemente Aldebaranium und Cassiopeium*, *Zeitschrift der Anorganischen und Allgemeinen Chemie*, **1910**, 67, pp. 102-106.
79. *Chemical Monthly*, **1913**, 84, 1713-1728.

80. There are hints in literature that point to the results also having been published in July 1913 in the proceedings of the Imperial Academy of Sciences Sect. II b, **1913**, 22. This seems likely as Carl Auer von Welsbach generally published his publications in the Academy first. However, these proceedings cannot be found today in the library of the Austrian Academy in Vienna.
81. See online: <http://www.ciaaw.org/atomic-weights.htm> (02/04/2019)
82. Eugen Schmahl, Carl Auer von Welsbach, *Abhandlungen und Berichte (Deutsches Museum)*, **1952**, 1, p. 4.
83. See Online: <http://www.chemieunterricht.de/dc2/lanthan/trennung.htm> (11/11/2018)
84. George von Hevesy, *Die Seltenen Erden Vom Standpunkte Des Atombaus*, **1927**, pp. 95-101.
85. Carl Auer von Welsbach, Über die chemischen Untersuchung der Actinium enthaltenen Rückstände der Radiumgewinnung, *Announcements of the Radium committee of the Imperial Academy of Sciences, part 1*, **1910**, 6, pp. 1011-1054.
86. Kurt Rossmannith, Fraktionierte Kristallisation von Ammonium- und Magnesiumdoppelnitrat der Ceriterden“, *Chemical Monthly*, **1995**, 126, pp. 543-547.