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GEOMETRY AS A PERSISTING BACKBONE METALANGUAGE IN ARCHITECTURE

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Albeit Architecture is not mere Geometry, much of Architecture is about Geometry. Either in real shapes, or in representational models, or in theoretical terms. Three simple questions should suffice to illustrate the point. What would a cathedral be, without its geometric feature? What would a BIM model collapse in, without its geometric framework? What would architectural theory be based on, without its reference to geometry of space? That is, looking through an ideal infrared lens, Geometry is revealed everywhere in the architectural field, from the dawn of time, and throughout the different technological eras. And not only in the architectural field, as René Descartes officially highlighted, establishing Geometry at the base of the nexus between logical (λόγος) and visual (γραφή) thinking[1], in the last decades widely reconfirmed, also at the computational test. This dual nature, making Geometry working as a metalanguage[2], so beneficial in agglutinating the multi-disciplinary field of Architecture, is even more to be considered when facing the challenges promised by AI (fig. 1), either in research or in the professional world, and especially in Geometry education, which is still our priority mission. Then, a further question seems to emerge. What would AI based architectural design be without Geometry? This paper aims at discussing some of the implications related to these topics, including some simple generative tests made with Midjourney and ComfyUI, and an instant tip from Chat GPT.

Keywords: Geometry and Architecture, Geometry as Metalanguage, Architectural Geometry and AI, Geometry Education in Architecture.

Introduction

Preparing to answer the editorial call from the point of view of Geometry, it seemed appropriate to look at its nexus with the other two key words there proposed, that is, Shapes, and Models. As a smooth start, the three synthetic definitions have been taken from the online Oxford dictionary. There, Shape is defined as «the form of the outer edges or surfaces of something», or as «an example of something that has a particular form». The first statement is intuitively closer to Geometry. In fact, even considering the mere appearance, Geometry is inside any outer edges or surfaces of something, characterizing it even independently on the material feature of that something. That's what makes the second statement true as well. Indeed, the independency on the material feature, allows to create examples or replicas with similar appearances. Including graphic replicas, that is, images, and graphic representations. This also confirms our initial question: what would a cathedral be, without its geometric feature?
About Model, it is defined as «a copy of something, usually smaller than the original object», which is the most common meaning. Differently from Shape, the definition seems more comprehensive here, since emphasis is not only on outer edges or surfaces. In the light of the shift from the Analogue to the Digital, the semantic density of the word Model became increasingly clear. With reference to digital representation, the Model has gradually moved away from the idea of a pure shape, getting in change the ambition to become a clone, that is, a digital clone of the real, by incorporating plenty of further information and parameters. Which seems to confirm our second question: what would a BIM (or similar) model collapse in, without its geometric framework?

1 The words λόγος and γραφή, literally 'logic' and 'graphics', refer to our two basic language roots, at least in our Western culture, as the reference to the Greek lexicon would emphasize; further theoretical insights can be found in Ugo, Lógos/Graphé.
2 That is, an artificial language, usable to operate on other artificial languages, and the related disciplinary fields; indeed, Geometry is used in several scientific, technical, and artistic areas.

Grouping the answers to the two questions mentioned above, we may say that, as well no cathedrals may be such without geometry, as any informed architectural model would collapse in lists of data without a geometric structure: in both the cases we would have, let's say, flesh with no bones without Geometry.
It seems time now to report how Geometry itself is defined in the dictionary, that is, «the branch of mathematics that deals with the measurements and relationships of lines, angles, surfaces and solids». In other words, a basic theoretical and operational system of intangible entities and properties, since Descartes established, supporting both visual and logical thinking.
According to Michel Serres[3], in the field of natural sciences and technology Geometry has a role comparable with law in social sciences and politics: it is everywhere. Back to Architecture, the relevance of Geometry is even more crucial, considering the impact of space in architecture, as outstanding masters have clearly highlighted, think of August Schmarsow, Nikolaus Pevsner, Bruno Zevi, or experimented, think of Peter Eisenman, Frank O. Gehry, Zaha Hadid, only to mention a few[4]. And Geometry is, in a way, the science of Space. To refer to the previous keywords, it offers architects theoretical foundations enabling to recognize, conceive, think of, and create, *Shapes* and *Models*. In this sense it works as a metalanguage, and at the same time, as a meta-structure. Which gives meaning to our third question: what would architectural theory be based on, without its reference to Geometry of space?

Geometry in Representation for Design Driven Research

As architects, our main task, mission and hope, is architectural design, construction, and maintenance. Which means, dealing with theoretical elaborations and (mostly) visual representation, be it carried out on paper, through a screen or immersive simulations. And physical mock-up, not included in this discussion. But what is the relation between project and research? Generally speaking, we can admit that any project is born from a specific research process, since it is based on a balance between problem-solving and creativity, traditionally related to the three Vitruvian categories at least, that is, to tangible and intangible requirements.

1 | *An image appearing on the web upon the prompt: Architectural Geometry and AI (Google: Jan 15, 2025).*

But a design driven research, needs wider bases, either theoretical or operational, not deduced from random single cases, but elaborated by a critical consideration of the history, as well as of the architectural contexts and traditions, with a certain systematic eye.
The fact is that a comprehensive theory of architecture is hard to define, not only because of the complexity of the field, but also because there is no objective "verity" in architecture, i.e. neither a Baroque nor a Renaissance church are "truer" than a Romanesque one[5].
However, complexity is also present to help us in this discussion to find a reasonable focus. What series of experts sitting around a table, appointed to promote a project, like engineers, sociologists, stakeholders, and the client, do expect from an architect? They certainly do not expect s/he to be more expert than them in engineering, sociology, finance, and so forth. They only expect, and require, that the architect understands all the various relevant inputs collected, evaluating, mixing and balancing requests and suggestions, and then turning them into an architecture, which means, into an architectural space, that is, shaped according to a specific and unique geometric configuration, suitable for the scope and appropriately integrated in the context[6].
Therefore, Geometry comes into question since the elaboration of the early

> *What would architectural theory be based on, without its reference to Geometry of space?*

3 Serres, *Les Origines de la géométrie*.
4 The theoretical position about space in Architecture is masterfully summarized in the well-known now classic book: Zevi, Architectura in nuce; also interesting is the book on Peter Eisenman by Pippo Ciorra, because of the interest of Eisenmann in space, who worked either as a professional or as a theoretician, across the analogue and the digital eras.
5 Architecture, indeed, is not like pure Science and Techniques, that is, new visions do not necessarily make the traditional ones obsolete, neither architectural trends are necessarily the same all over the world, despite the globalization.
6 Cocchiarella, *La "forma" oltre il "codice"*.

2 | *Skizoid, by Joris Putteneers. (Hovestadt 2020, p. 173).*

informative recognisance on the project site, as well as during the entire design process, and finally in the built shape, including its life cycle. It offers a valuable metalanguage supporting any step of the design process, starting from setting the topology until to achieve the final configuration[7]. In other words, the architect can take advantage of the creative potential offered by the several *transformations* provided by Geometry[8]. In the glorious past it basically was Euclidean Geometry, and particularly symmetry, as the classic treatises and the coeval built architecture confirm, but with the advent of the modern Mathematics, new frontiers have been opened[9], and in fact, we talk about Geometries nowadays, which are integrated in the Digital Graphics environment (fig. 2).

It is clear at this point, that a design driven researches should include a focus on the geometric strategies supporting the whole design process, that is, not only scientific literature and erudition, but appropriate geometric background, practice, and sensitivity. This means a complete overturning of the popular opinion about the *deductive* approach to Geometry, encouraging at the same time experimentations in the direction of an *inductive* approach, addressed to the development of projects[10].

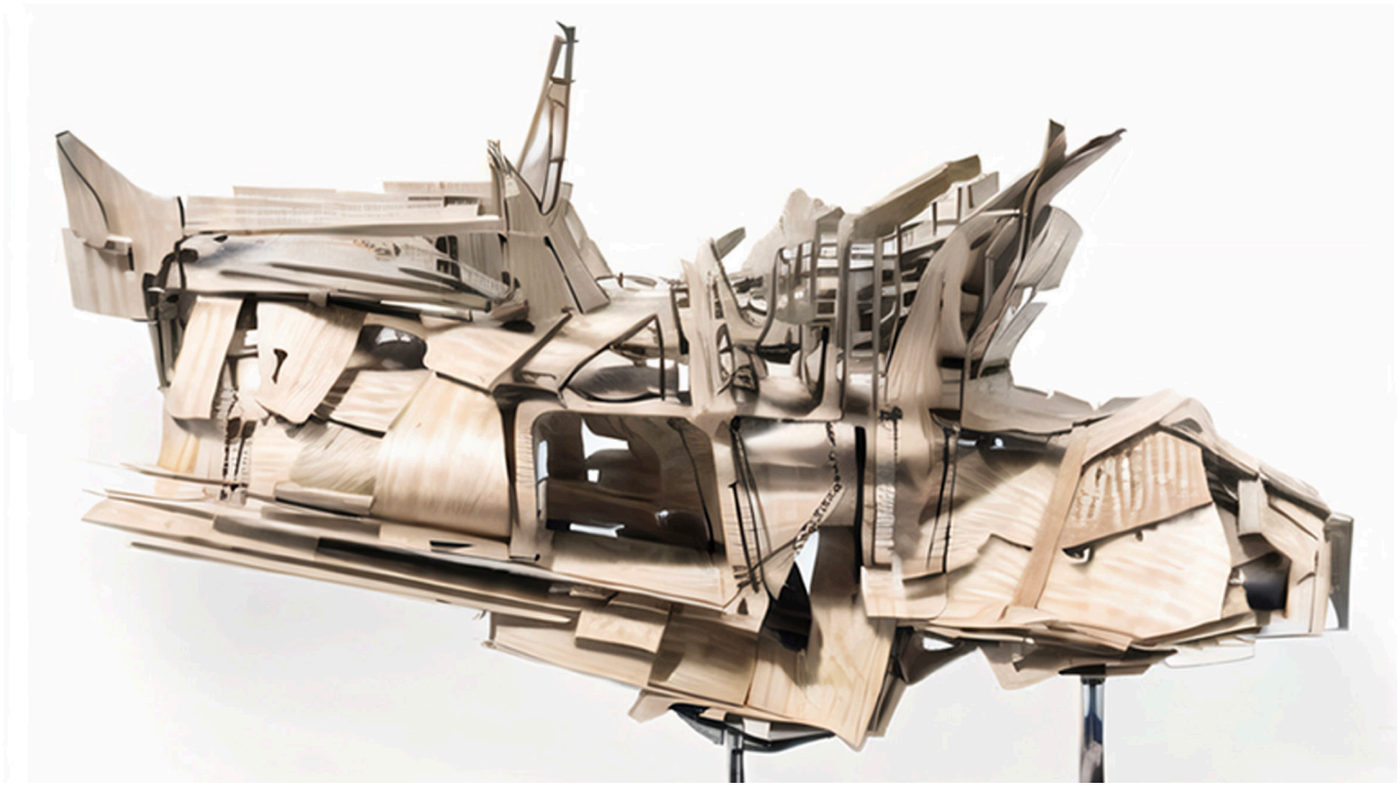
On closer inspection, this trend had a long tradition, since the origins of architectural drawing, touching an apex in the Renaissance and Enlightenment treatises, where geometrical patterns widely "stand for" architectural compositional guidelines. More recently this trend was also confirmed with respect to the modern Geometry, inspiring architects to envision revolutionary changes, and new theorizations.

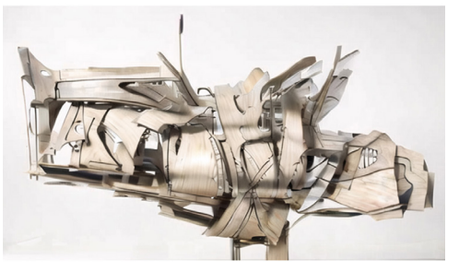
Structuralism first, to which some milestones can be referred, like *The Geometry of Environment*, by Lionel March and Philip Steadmann, where a rich series of projects is analyzed, comparing them with the geometric transformations laying behind their composition[11]. Looking at this work nowadays, it shows that everything was ready for the advent of the Digital. A bit later historical retrospect belonging to the Analogue era can be also mentioned, *The Projective Cast* by Robin Evans, where the use of Geometry in architecture is discussed, remarking in particular its active power in supporting architectural thinking, imagination, drawing, and building, even at the higher semantic levels of architectural aesthetics and rhetoric[12].

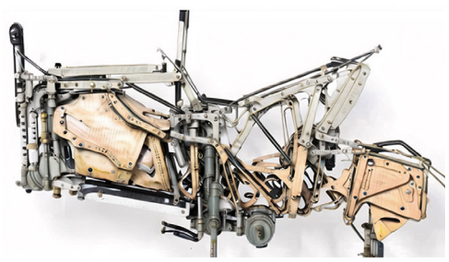
In more recent times, just over two decades, two contributions may be mentioned, respectively from a mathematician and from an architect.

The first one is a well-known compendium of geometric structures suitable for architecture, revisited through a digital approach, by Helmut Pottmann et al., titled *Architectural Geometry*, where the

7 Hemmerling, Cocchiarella, *Informed Architecture.*
8 Glaeser, Polthier, *Bilder der Mathematik*
9 Boyer, *A History of Mathematics.*
10 Cocchiarella, *Geometry and Graphics in Spatial Invention: Among Mind, Hand, and Digital Means.*
11 March, Steadman, *The Geometry of Environment: An Introduction to Spatial Organization in Design.*
12 Evans, *The Projective Cast.*







3 | *An example of Semantics-driven morphing. AI driven transformation process of a geometric structure, leading to different solutions, like a potential space (small image above) or like a potential mechanism (small image below). Discarding mechanism and evolving in the direction of a space, the configuration in the big image at the top of the page starts showing a gate, maybe promising as the entrance of an architectural space. (Civitai, Feb 21, 2025; graphic composition by author).*

geometric transformations reported are often compared with real cases including contemporary architectural examples[13]. The second book, *Beyond the Grid,* by Ludger Hovestadt, is an interesting compendium of experimental tests and research projects, strongly geometry based, carried out in the field of parametric modeling for architectural design and digital fabrication during about twenty years at the ETH Zürich[14]. Especially in this latter, thanks to the computational approach, Geometry is shown as a living plastic discipline in the hands of an architect, and its contribution appears as no longer separable from the design process. Like a modern treatise, connecting and integrating theoretical elaboration and experimentation, it shows a valuable way to use Geometry as a metalanguage in design driven research in architecture. The next step concerns some notes relating to our fourth and final question: what would AI based architectural design be without Geometry?

Architectural Geometry and AI

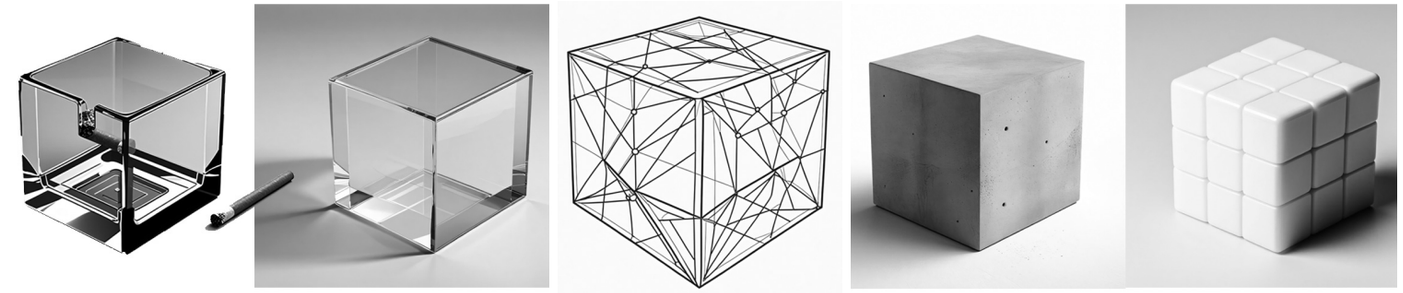
Although the early experimentations in the direction of AI date back to the Forties of the Twentieth century, it is only at the beginning of the Sixties that researches on *scene analysis* by artificial vision through machines had a start, appearing on the PhD dissertation of Larry Roberts at the MIT. This happens just a couple of years after the *sketchpad* was invented by Ivan Sutherland at the same University, giving birth to 3D space modeling. To carry out experiments on scene analysis, a virtual "micro-world" was required, populated by virtual objects, which appeared after a while, in 1971, thanks to David Huffman. Other versions of virtual spaces realized to test artificial vision showed a room with squared tiles, a model very similar, in its appearance, to those used in the early examples of perspective constructions during Renaissance.

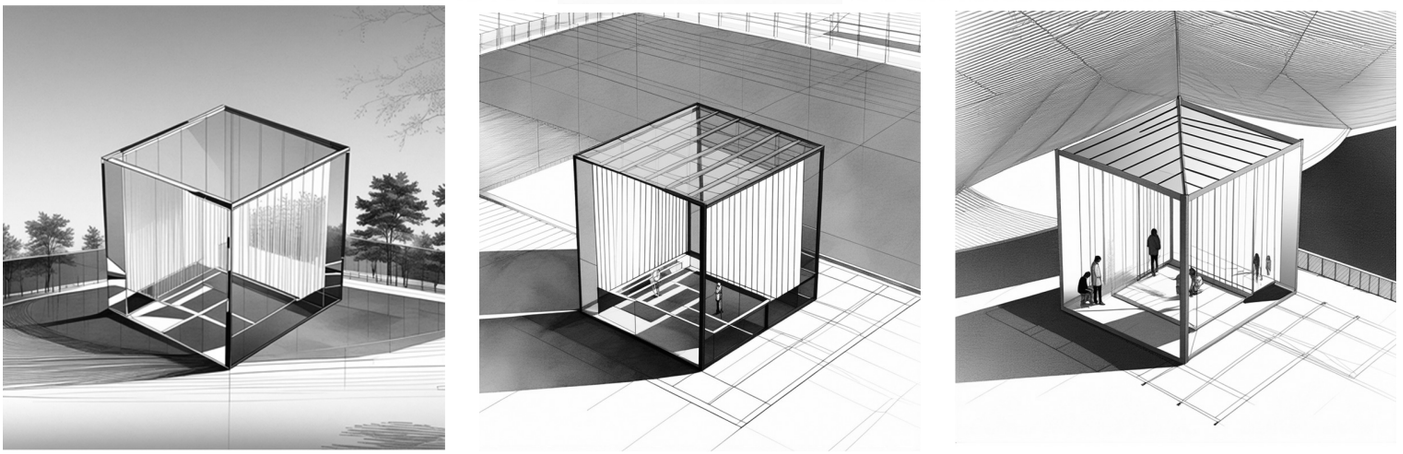
It was an interesting turning point in the field, since finally Geometry started appearing on the surface of the Digital, becoming visible on a visual interface.

AI, as any digital based or co-based system, has two sides, the *backend* and the *frontend*, the first one concerns how it works, the latter concerns the user interface, that is, the tools we can operate with, which is our point of interest[15].

As it has been reminded by Nils J. Nilsson[16], over time AI got benefits from joining the so called *symbolic* elaboration, based on top-down processes related to prior knowledge recorded as sets of data, with a human operator as the mediator between machine and real context, translating information into inputs; and the so called *sub-symbolic* elaboration, based on

13 Pottmann et al., *Architectural Geometry*.
14 Hovestadt, *Beyond the Grid*.
15 Quintarelli, *Intelligenza artificiale*.
16 Nilsson, *Artificial Intelligence: a New Synthesis*.





4 | *A generative test carried out with ConfyUI. The main prompt (a cube) has been integrated with further targeted inputs, to obtain a crystal cube, a crystal ashtray, a concrete cube, a plastic cube (resulting in a kind of monochrome Rubik cube); below, options resulting upon the addition of further specific inputs to obtain a pavilion; above, midst, what ComfyUI shows as a 'pure' geometric cube (Elaboration by author, with Matteo Cavagliá).*

5, 6, 7 | *Generative tests caried out by Midjourney, with the following prompts and no other inputs: crystal cube; cubic crystal ashtray; 5 storey cubic building equipped with zero energy impact technologies. Sometimes the cube is considered in a 'broad' sense (Elaboration by author, with Matteo Cavagliá).*

8 | *Generative tests caried out by Midjourney, with the following prompts and no other inputs: architectural pavilion with a cubic shape and crystal-like surfaces and people inside. Again the cube is intended in a broad sense, and the size is not precisely fixed (Elaboration by author, with Matteo Cavagliá).*

bottom-up processes, aiming at sensitizing the machine to signals as well, without prearranged symbolic transcriptions, by the direct interaction with the context around through sensors, in order to build up fresh knowledge on the basis of the info memorized, that is, of an artificial experience based way, somehow following the natural learning processes, which is still receiving great impulses from the advancements in neurosciences[17].

In fact, the most recent AI systems can learn not only from codified knowledge, but also from imitating our mental processes, as they appear recorded on pictures, videos, and information, provided by neuroimaging, and related data mining. According to Stuart Russel and Peter Norvig[18], joining *symbolic* top-down and *sub-symbolic* bottom-up machine learning strategies, opened the way to combine parametric and non-parametric elaborations, in the aim of having at the end an *Intelligent Agent*, able to react and adapt to new contexts, instead of a "simply" intelligent system.

It means that the most advanced mathematical AI engines can nowadays rely on the so-called *neuro-symbolic* learning processes, which have enormously increased their performative level compared to the former prototypes.

But Geometry is anyway in the body of the machine, and nowadays we can also talk about Geometry of deep neural networks, as it has been well described recently by Jong Chul Ye[19]. Moreover, in the opinion of some researchers, the (visual) graphic features of the geometric structures, are resulting of great help in simplifying the manipulation of the algorithms. Something reminding us the opinion of Einstein by the way, who used to eulogy the power of visual thinking.

More generally, the combination of *knowledge-based* and *experience-based* approach, led to a contamination, and a cross-fertilization of languages, and consequently, a reciprocal increase of functionalities and semantics[20].

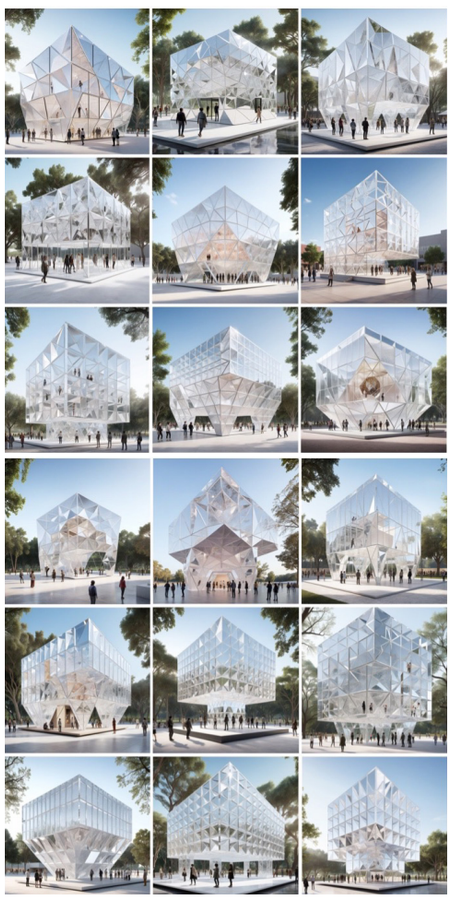
Concerning functionalities, they have generated new type of tools in the end. If the first way mentioned mainly refers to computers, and the second one mostly to robots, the combination of the two has in part empowered robots, and in part created kind of software robots, that is, software able to work as robots in search of specific info in the web. This is the case of the *Softbots*, as they have been defined by Oren Etzioni and Daniel Weld in the Nineties[21]. Chat GPT offers a good example of what they can do[22].

And here we pass to the next point, that of semantics, related to architectural geometry and AI. Far from the community of pure mathematicians, it is hard to find AI systems able to manage and generate pure geometric structures. Pure AI based Geometry is nowadays a rather specialized topic of interest for mathemati-

17 Dehaene, *How We Learn;* Mallgrave, *L'empatia degli spazi. Architettura e neuroscienze*; Robinson, Pallasmaa (eds.) *La mente in architettura. Neuroscienze, incarnazione e il futuro del design.*
18 Russel, Norvig, *Artificial Intelligence: a Modern Approach.*
19 Ye, *Geometry of Deep Learning.*
20 Hovestadt, Hirschberg, Fritz (eds.), A*tlas of Digital Architecture.*
21 Etzioni, Weld, *A Soft-Box Based Interface to the Internet*.
22 Indeed, Chat GPT works as a virtual robot in the web, elaborating on the specific task assigned via the prompt.
23 https://en.wikipedia.org/wiki/AlphaGeometry.
24 del Campo (ed.), *Artificial Intelligence in Architecture.*
25 https://www.promeai.pro/.
26 https://architechtures.com/en.

cians. With reference to Euclidean space, among others, *Alpha Geometry* may, for example, offer a good picture of how AI systems for Geometry work[23].
Naïvely, one can think that this type of AI driven technologies may allow architects to push the boundaries of Geometry, by generating a great number of multiple spatial options, to be selected and adapted to a specific project afterwords. As it was in the past, when using symmetry or parametric modeling first, and then testing whether they could fit for the ongoing architectural projects.
Given that we keep ourselves sufficiently up to date to interact efficiently with the machine, or, unless completely novel approaches will emerge[24].
In most cases, in fact, the combined elaboration of symbols and signals has made machine learning able to deal at the same time with several types of languages, i.e. logic and iconic, either in the *backend*, or in the *frontend*. As a result, thanks to their enriched semantic power, the current AI systems allow us to input statements through the interface, receiving statements, numbers, sounds, signals, images, movies in turn. Or, we can input numbers, sounds, images, or signals.
Which has *pros* and *cons*. This languages shift, indeed, like asking for images by words, etc., may open the way to misunderstandings, but the advent of graphic interfaces is making the job easier to control, at least with reference to our field.
And the potential advantages are indisputably invaluable, since AI is designed to work across languages, providing in turn valuable channels for interactions among different disciplinary fields.
The geometric configurations elaborated by the AI systems conceived for architecture, in fact, are not simply resulting from abstract transformations, but they may be sensitive to various other conditions pertaining to architecture itself, like stability, functionality, environmental performances and sustainability, to mention a few, including the human operator, based on which Geometry adapts automatically.
Some generative design tools specific for architecture are available, like *PromeAI*[25], or *Architechtures*[26], and several others, where design simulations are proposed in real time, and the Geometry is perfectly consistent with that of a real architecture.
Since AI works with the enormous database of the Internet, its complex semantic structure attracts series of data potentially related to the task, which makes it

ready to elaborate the output in several direction. In figure 3, three images of an elaboration process, found on *Civitai*[27] are presented (fig. 3), where a geometric configuration may evolve in the direction of a space (above), or, of a mechanism (below). In any case, apart from the specificities related to the other parameters, what brings together the frames in this process is clearly related to their geometric structure, that is, the skeleton of the ongoing transformation.
The prospective solutions offered by AI can furthermore be related to, or predict, specific "scenarios", taking advantage of *serious game* environments, where not only space, but also the parameter time is involved and various immersive levels of fruition and interaction can be set[28].
Speaking of interaction levels, in order to checking the difference, we made similar tests with *ComfyUI*[29], which allows the use of prompts together with further customized inputs, and with *Midjourney*[30], using single prompts.
Although in the first case the output seemed to be better oriented to specific solutions (fig. 4), while in the second the range of variants seemed more unpredictable (figs. 5-8), either *ComfyUI*, or *Mid-*



9 | *Synthetic data generated by Alpha Geometry (Google Deep Mind, February 28, 2025).*

journey, show a semantic density, resulting from the abundance of information available on the web, that is, they share the same data source, where the geometric features of spaces and objects are usually linked to other physical properties and information. Quite paradoxically, that of a pure geometric cube seems the hardest image to be elaborated, as shown in the figure[31].

We can then deduce that, given its flexibility, interesting collaborative perspectives are also to be fully explored, on design processes carried out by integrating AI systems and human teamwork, possibly encouraging interdisciplinary cooperation between architects and other professional profiles, including engineering, natural sciences, and humanities, since all these fields should be involved in providing the best possible *Genius Loci*[32] to the humans, through the architectural places.

All in all, in this case AI it aims at replacing the holistic approach of the human driven architectural design, leading to the architectural project as a syncretic outcome. We will see how things will evolve in the future.

Moreover, architectural fabrication and life-cycle assessment, maintenance, disassembly and disposal can be managed or predicted, which, as it has been said already, are not included in this discussion.

End with a Tip from AI

At the end of this short excursion, we may conclude that Geometry is confirmed, with all its branches, from Euclidean geometry to the geometry of Hyperspaces, as a backbone metalanguage for architectural representation and design, even in the AI era.

On the one hand, Geometry is also at the base of the algorithmic structure of the AI systems, while with reference to architecture, traditional and computational geometric *shape grammar*[33] seems nowadays enriched with new informative components, allowing more holistic approaches to the project.

In particular, AI based architectural Geometry is not necessarily to be considered in terms of pure abstraction (fig. 9), but it can be sensitive to plenty of other parameters, related to the tangible and intangible aspects, specifically characterizing architecture and its feasibility.

This kind of *syncretism* facilitates a collaborative and multidisciplinary approach to architectural design, also thanks to shared visual interfaces. In addition, the ease in getting real time AI reactions to inputs, encourages to explore *forecast scenarios*, which is in line with what in the tradition would have defined in terms of search for the *Genius Loci*.

In this perspective, information embedded in the AI based models somehow fills the gap, at least at a representational level, between the abstract design of a geometric *space* and the identity of a real *place*. The *informative density* of the AI based architectural design outcomes, finally brings us back to the three terms proposed by the call: Shape, Models, and Geometries, all three included and interacting in this novel process.

It is clear that besides research and professional experimentation, we also need to review education, and probably a new mentality to appropriately work with AI, including it as part of our background, without losing our identity.

AI seems to agree with us, given that, when asked about Geometry education in architecture in the AI era, Chat GPT wisely answered: «In the AI era, geometry education in architecture must adapt to incorporate both traditional principles and modern technological advancements. As AI and computational tools increasingly shape architectural practice, architectural education needs to strike a balance between understanding classical geometry and mastering new digital methods of design and analysis. Students must develop a strong foundation in geometry to understand form, space, and structure, while also gaining proficiency in digital tools that leverage AI to push the boundaries of geometric exploration»[34].

A short note may be added to our discussion, concerning the worry about *human authorship*, if it might be dethroned by AI. At the present stage of technology, it does not seem it may happen, since AI is mainly used to react to human inputs, at least in the architectural design process. Instead, the intrinsic "imperfection" of the humans, who are limited by their own non-standard experience, personal background, individual sensitivity, and reaction to stimuli, can even displace and surprise AI, which works on much wider, but standardized bases. These are only some of the challenges expected in the disciplinary field of *Geometry* related to *Architectural Drawing* nowadays, whose affinity with the wider world of *Representation* is, anyway, confirmed and even more clear in the era of AI.

27 https://civitai.com/.
28 Baricco, *The Game*.
29 https://www.comfy.org.
30 https://www.midjourney.com/.
31 The number and the type of tools for AI is constantly growing, as well as the possibility to make their interaction an essential part of the machine learning process.
32 The concept has been explored in depth by Christian Norberg-Schulz, in the milestone book Genius Loci, Towards a Phenomenology of Architecture Rizzoli, New York 1980.
33 https://en.wikipedia.org/wiki/Shape_grammar.
34 Access: January 15, 2025.

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