ARITHMETIC, GEOMETRY, AND MEASUREMENTS IN THE BAROQUE BUILDING SITE

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Between the 16th and 17th centuries, intellectuals, mathematicians, and military engineers wrote treatises dedicated to the measurement of architecture and fortifications, while the practice of surveying was considered a nearly professional endeavor.

Modo di misurare le fabriche (1674) by Guarino Guarini responds to the need for expeditious methods for measuring architectural elements in buildings under construction during the second expansion of Baroque Turin. Dedicated to the measurement of surfaces and volumes, unlike other texts, the treatise devotes very little space to the description of surveying techniques. The work stands out for its absolute novelty in the treatise landscape, as it refers to the construction phase of the architectural process.

In this essay, new considerations are developed regarding the relationships between arithmetic, geometry, and measurements, highlighting their connections to the construction of buildings.

The analysis carried out underlines the need to read Guarini's writings on architecture as a complementary system of highly integrated knowledge. In particular, the link between geometry and architecture, intrinsic to Guarini's treatises, is expressed in Modo di misurare le fabriche through the concept of measurement.

Keywords: Survey, Measurements, Geometry, Guarini, Modo di misurare le fabriche.

Introduction

Over the centuries, surveying and measurement have revealed different possible interlacings.

Surveying, as a technique that uses instruments to acquire angular and linear dimensions, interacts with measurement depending on the objectives of the survey and the characteristics of the object under examination, whether it be, in the scope of built heritage, an urban fabric, a building, or a detail. In turn, measurement characterises the object from the design stage, defining its modularity and proportions.

Between the 16th and 17th centuries, intellectuals, mathematicians, and military engineers wrote treatises dedicated to the measurement of architecture and fortifications[1], while the practice of surveying was considered a nearly professional endeavor[2].

Modo di misurare le fabriche (1674) by Guarino Guarini, who was an abbot of the Theatine order, fits into this context, responding to the need for expeditious methods for measuring architectural elements in buildings under construction during the second expansion of Baroque Turin. Dedicated to the measurement of surfaces and volumes, unlike other texts, the treatise devotes very little space to the description of surveying techniques. One hundred thirty-one pseudo-axonometric diagrams, produced using xilography[3], illustrate the text propositions. This essay, which builds upon previous studies on the calculation of vault surfaces in the treatise. It develops new considerations on the relationships between arithmetic, geometry, and measurements, highlighting their interconnections with the building's construction.

[1] Vagnetti, *La teoria del rilevamento architettonico in Guarino Guarini*, pp. 503-505.
[2] Docci, Maestri, *Storia del rilevamento architettonico e urbano*, p. 136.
[3] Scotti Tosini, *Testo e immagini nell'Architettura civile e nelle opere teoriche di Guarini*, p. 96.
[4] Vagnetti, *La teoria del rilevamento architettonico in Guarino Guarini*, pp. 497-511.
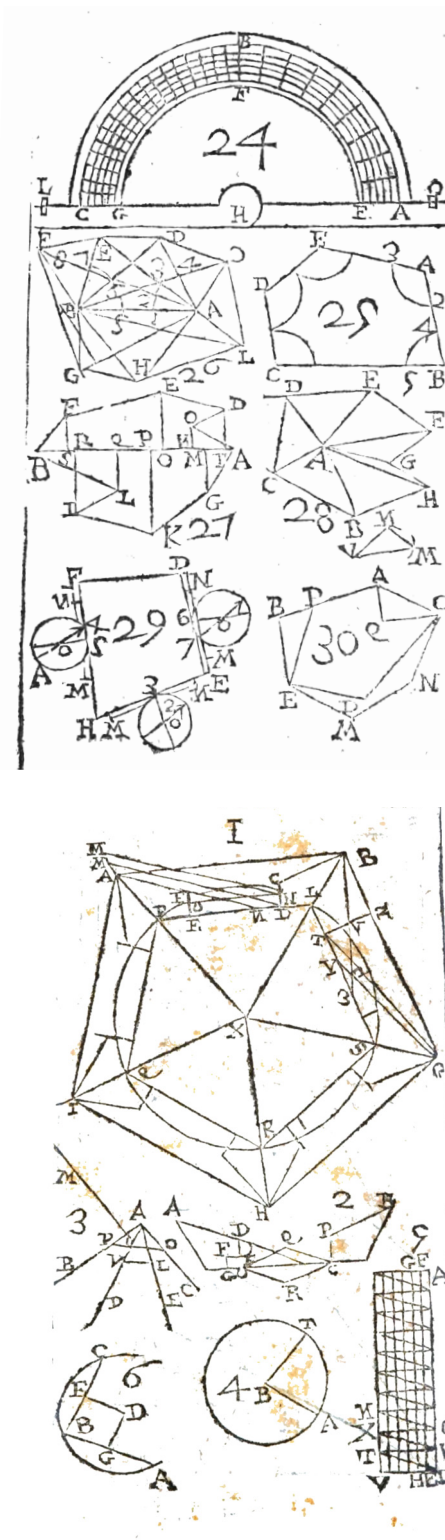[5] Docci, Maestri, *Storia del rilevamento architettonico e urbano*.
[6] Vagnetti, *La teoria del rilevamento,* cit., p. 500.
[7] Tavassi La Greca, *La posizione del Guarini in rapporto alla cultura filosofica del tempo*, pp. 452-453; Bianchini, *La scienza della rappresen-*

1 | *The mobile square and the different techniques for using the instrument. Trattato di fortificatione 1676, plate 5.*

2 | *At the bottom right, the composed graphic scale for reducing measures in trabucchi and piedi liprandi. Source: Trattato di fortificatione 1676, plate 1.*

tazione nella concezione di Guarino Guarini, pp. 22-41; Spallone, *Rappresentazione e progetto. La formalizzazione delle convenzioni del disegno architettonico*, p. 58.

8 Roero, *Guarino Guarini and Universal Mathematics*, p. 418.

Survey and measurement in the Baroque era

Luigi Vagnetti[4] and Docci and Maestri[5] provided a comprehensive overview of the state of the art in surveying during the Baroque age

Vagnetti recalls the entire Renaissance tradition of surveying, closely linked to the rediscovery of antiquity, which had its forerunners in Brunelleschi, Donatello, and Alberti, culminating in Raffaello's proposal described in his Letter to Leo X and in Palladio's systematic survey campaigns. The author sharply observes both the conceptual distance between the recognition and understanding of the formal lexicon of antiquity achieved in the 15th century and the need to account for the works built, which is the motive aim of Guarini's work. Moreover, he highlights the absence of methodological and regulatory texts suitable for governing survey activity until the late Renaissance, so that we can only appreciate the graphic results. Finally, Vagnetti emphasises the novelty of the Baroque master's work: no one before him had ever dealt with the method of measuring the surfaces and volumes of architecture[6].

Docci and Maestri observe the continuity of techniques and practices in the 17th and 18th centuries with respect to the two previous centuries.

Still, they also note the different ways of conceiving culture and science, particularly in the field of surveying. Measurement techniques were perfected, and representation systems received greater attention with respect to the codification that would lead to their formalisation through the work of Monge. The surveying of monuments assumed a general knowledge function aimed at achieving greater objectivity. The practice of surveying almost took on the status of a profession, with artists and architects conducting architectural and urban surveys for educational purposes, aided by advances in xylography and chalcography, lower paper costs, and the widespread use of printing.
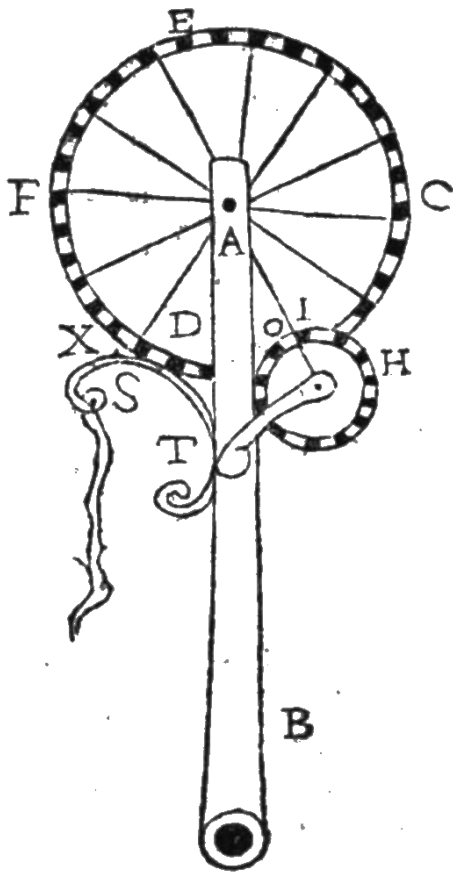
Furthermore, we must not forget the significant boost provided by the territorial surveys that became necessary from the 16th century onwards in the field of fortifications.

In fact, the transformation of war techniques brought about by the use of new weapons that employed gunpowder led to the need to build "modern" fortified structures, often by modifying or adding to existing buildings, or by creating them *ex novo*. The intertwining of geometry, architecture, and ballistics, necessary for the new techniques of city defense, required very accurate surveys, translated into precise plans.
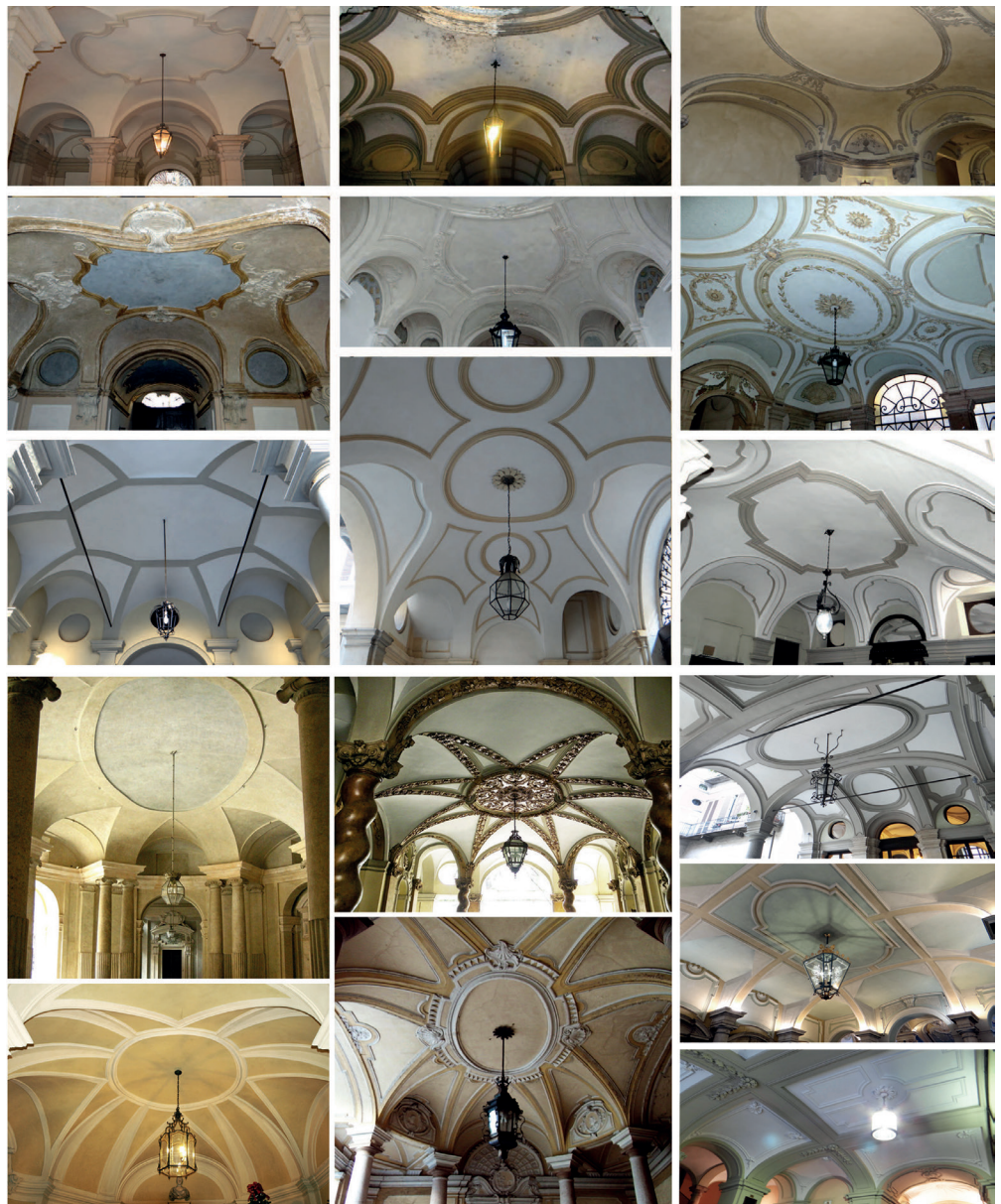
Survey and measurement in Guarini's treatises

The thematic complementarity between Guarini's theoretical works on architecture has been highlighted in several studies[7]. In *Modo di misurare le fabriche*, the Theatine constantly refers to his previous extensive work on geometry, *Euclides adauctus et methodicus mathematicaque universalis*, published in Turin in 1671 by Bartolomeo Zapata. Scotti Tosini and Roero have highlighted the relationship between the two treatises[8]. a central topic, *Modo di misurare le fabriche*, namely the measurement of the surfaces and volumes of vaulted structures, the main contributions of Euclides are evident. They come from *Tractatus XXIV*, which deals with conics, *Tractatus XXXI*, which is related to the calculation of surfaces, and *Tractatus XXXII*, which can be linked to stereotomic techniques through the illustration of intersections between solids, between solids and planes, and developments on the plane. Furthermore, from the dedication to the readers, Guarini recalls that in Euclides, he had illustrated methods previously unknown for measuring surfaces and solids, but without specifying their applications to the measurement of buildings, as he would do in *Modo di misurare le fabriche*. The actual references to architecture in the latter text will be discussed in the following paragraphs. Still, the fact remains that Euclides is constantly mentioned in the development of each topic and that some figures coincide, suggesting that Guarini reused the duplicate drawings in the exact dimensions in both works[9].

Equally interesting are the connections with the other two works dedicated to architecture and construction: the

3 | *Territorial surveying methods using different techniques and instruments.(From G. Guarini, Architettura civile 1737, plate 1, treatise II).*

cant when fortifying an *ex novo* structure or when modifying an existing fortified structure to enhance its defensive capabilities. The tools and techniques proposed help measure angles and lengths. In the first case, Guarini describes the construction of the mobile square (fig. 1), an instrument with two arms, one fixed and one mobile, mounted on a graduated half-circle, which allows the angles to be read in sexagesimal degrees. In the second case, he illustrates how to take linear measurements, also using ropes and poles. More space devoted to the graphic representation of the measurements on paper. To carry out this step, it is essential to construct a composed scalebar (fig. 2) for the proportional reduction of the actual measurements[10], which allows for the measurement of *trabucchi* and *piedi liprandi*, the measurement units used in Piedmont at that time (one *trabucco* is about 3.0825 m, one *trabucco* contains six *piedi liprandi*).

The mobile square can therefore be used to fix lines perpendicular to an axis, to construct trilaterations, to fix lengths with respect to two vertices, and to draw angles at the vertices of polygons. These graphic tracing choices depend on the site's features and are directly related to the peculiarities of the ground in terms of intervisibility between the vertices to be projected onto the plane.

In *Architettura civile*, the discussion of surveying is located in *Treatise II*, dedicated to icnography, i.e., the orthogonal projection of civil buildings in plan. In *Treatise II*, the topic of building design in plan, guided by geometry. Guarini's definition of icnography as «a description on paper of the buildings, from which, in the plan where they are to be constructed, measurements are taken to locate the building»[11]. The discourse continues through the framing of a real workflow, still common today in direct architectural surveying, which involves in the operations of leveling, measuring, and drawing, the construction of a scalebar in accordance with local units of measurement, and the application of the methods and techniques of representation.

The description of the surveying instruments is quite detailed: a ruler with a plumb line, a spirit level, a ruler with a

Trattato di fortificatione of 1676 and the *Architettura civile*, published posthumously in 1737.

In his *Trattato di fortificatione*, ch. 2 - *Del modo di levar'un sito per fortificarlo* (On the method of surveying a site for fortification), located within Book II - *Delle fortezze irregolari* (On irregular fortresses), Guarini raises the issue of metric surveying of the territory. As mentioned above, this issue is particularly signifi-

9 Scotti Tosini, *Testo e immagini nell'Architettura civile*, cit., p. 96.

10 Bevilacqua, Spallone, *Composed Graphic Scales in the European Military Treatises and Manuals from the 17th to the 19th Centuries*, pp. 180-189.

11 Guarini, *Architettura civile*, p. 38.

mirror for leveling, a rope, poles, and a telescope for constructing alignments, a square and a mobile square for measuring angles constitute the surveyor's toolkit. Angle measurement, in particular, is carried out with the mobile square placed at one, two, or more station points; the instrument can be replaced with a compass and magnet.

Great attention is paid to the units of measurement in use in different geographical areas, with the possibility of conversion between them, by the comparative scheme of seven scalebars. It refers respectively to the piede liprando, the King's or geometric foot, the Roman foot, the Cremonese arm, the modern Roman palm, the Spanish foot, and the Venetian foot, which, through subdivisions and comparisons, identify additional units used in other places. Compared to the *Trattato di fortificatione*, the development of surveying techniques, again on a territorial scale, in *Architettura civile* is more comprehensive, extending the discussion to levelling and the comparison between units of measurement, with the presumed aim of fully explaining the techniques of planimetric graphic restitution (fig. 3).

The *Modo di misurare le fabriche*

Modo di misurare le fabriche was published in Turin in 1674 by the Gianelli heirs. At that time, Guarini was living in the city, having been called there in 1666, first by the Theatines, and then commissioned to design civil and religious buildings, as well as to teach some of the descendants of the Savoy family. His stay in Turin ended with his death in 1683.
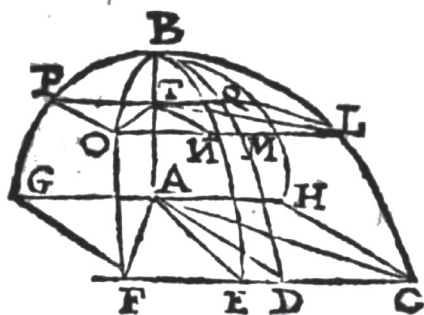
The treatise is compact in size, measuring 10.5 x 18.5 cm for ease of handling. It is written in Italian and consists of 208 pages.

Pseudo-assonometric diagrams, probably not autographic, illustrate most of the propositions.

The volume is dedicated to Giovanni Andrea Ferrari, Count of Bagnolo, Presi-

4 | *Guarini's design of a toothed wheel for measuring the lengths of oval-section vaults. (From G. Guarini, Modo di misurare le fabriche 1674, p. 48).*

5 | *Complex vaulted systems in Baroque Turin. Photo by M. Vitali.*

6 | *Lowered pavilion vault on a rhombus plan. (From G. Guarini, Modo di misurare le fabriche 1674, p. 105)*
Angular pavilion vault in the street-facing atrium of Palazzo Provana di Collegno by Guarini.
Photo by M. Vitali.

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The intertwining of geometry, architecture, and ballistics, necessary for the new techniques of city defense, required very accurate surveys, translated into precise maps.

dent and General of Finance of His Royal Highness. Scotti Tosini states that the text was intended to provide clear rules in an area characterised by uncertainty and disputes, often linked to the financial fragility of builders caused by prolonged payment delays.

The specific aim, however, was to support the Count of Bagnolo in his supervisory duties as the city's financial manager during its intense period of transformation[12]. The dedication expresses the aim of providing methods for measuring the surfaces and volumes of buildings under construction during the second expansion of Baroque Turin, undertaken in 1673 under the guidance of Amedeo di Castellamonte. It is, therefore, a work with an explicit practical purpose, for the calculation on site of materials and works under construction, a sort of operational complement to Euclides' demonstrations. Vagnetti emphasizes the absolute novelty of this approach, also in relation to the practice of architectural surveying, which had been consolidated over 250 years of activity on ancient monuments[13]. An introductory prelude provides the basic knowledge in the field of arithmetic, which helps tackle subsequent calculations. These are, in particular, the four operations, including tests to verify their accuracy, the search for the proportional quarter, the calculation of the square and cube roots, and the multiplication of lengths in relation to units of measurement. The prelude continues with some additions to Euclides, presented as «propositions that supplement our augmented Euclides»[14] expanding the variety of surfaces considered. The prelude concludes with some indications and tips relating to surveying techniques and equipment, including the invention of a toothed wheel, a valuable instru-

ment for measuring the lengths of oval-section vaults (fig. 4).

The treatise is divided into three parts, dedicated respectively to the rules for calculating the areas of plane figures, the surfaces of solids, and the volumes of solids.

The first part is divided into 18 propositions contained in three chapters devoted respectively to the measurements of different countries (Chapter I), the measurements of plane surfaces with straight contours (Chapter II), and the measurements of plane surfaces with curved contours (Chapter III).

The second part comprises 35 propositions, developed across five chapters dedicated to calculating the surfaces of solid bodies. The chapters distinguish between flat and cylindrical surfaces (Chapter I), conical surfaces (Chapter II), pavilion vaults and lunettes (Chapter III), spherical and spheroidal surfaces (Chapter IV), and annular and spiral surfaces (Chapter V).

The third is dedicated to the measurement of volumes: 45 propositions are part of ten chapters dealing with the calculation of the volumes of solids bounded by flat surfaces (Chapter I), cylinders and cones (Chapter II), spheres, spheroids, and vaults (Chapter III), rectangular, parabolic, and hyperbolic conoids (Chapter IV), spheres, spheroids, and ellipsoids (Chapter V), parabolic and hyperbolic conoids (Chapter VI), annular solids (Chapter VII), spiral-shaped solids (Chapter VIII), solids bounded by curved surfaces (Chapter IX), and hollow solids (Chapter X).

The appendix contains quick solutions for calculating volumes and surfaces.

As we shall see, the list of chapters above only partially reflects the intertwining of strictly geometric themes with built architecture.

The treatise and architecture

Vagnetti states that the work is a manual in which architecture is mentioned almost in passing and further defines it as a normative text in which buildings, reduced to simple surfaces or regular volumes, can be described using the vocabulary of a mathematician, or at most a geometer, rather than an architect.
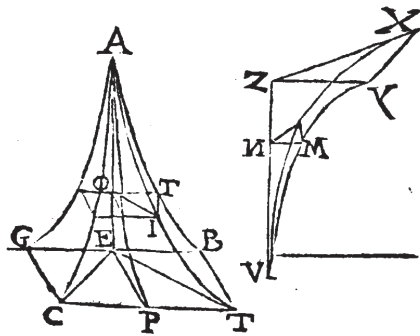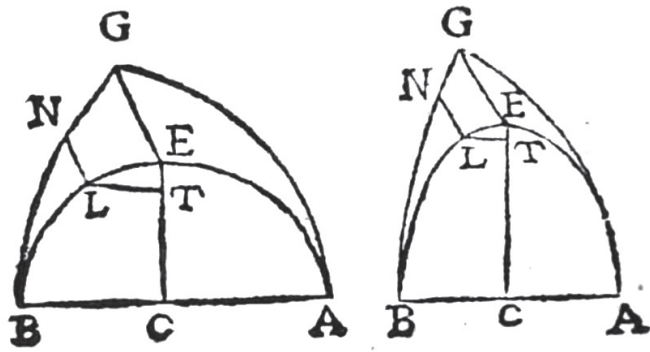
12 Scotti Tosini, *Testo e immagini nell'Architettura civile*, cit., p. 96.
13 Vagnetti, *La teoria del rilevamento,* cit., p. 500.
14 Guarini, *Modo di misurare le fabriche*, p. 33.
15 Vagnetti, *La teoria del rilevamento,* cit., p. 499.
16 Scotti Tosini, *Testo e immagini nell'Architettura civile*, cit., pp. 96-97.

7 | *Elliptical section groin. Source: Modo di misurare le fabriche 1674, p. 120. Groins having an elliptical section in the courtyard atrium of Palazzo Provana di Collegno by Guarini. Photo: M. Vitali.*

8 | *Pointed groin obtained from a concave pyramid. Modo di misurare le fabriche 1674, p. 122.*

The scholar supports his interpretation by conducting a careful examination of the treatise and highlighting passages where the measurement of architectural elements and parts is explicitly mentioned[15].

Scotti Tosini summarises Vagnetti's observations, highlighting the peculiarity of Guarini's method: in his words, «he starts from elementary notions and gradually moves on to more complex themes, focusing on the possibility of effortlessly measuring any artefact by reducing architecture to simple surfaces or volumes of regular shapes, comparable to those of the geometric vocabulary»[16].

However, it is necessary to emphasize that the links between architecture and geometry, which recur throughout Guarini's theoretical work, emerge from the very first pages of *Architettura civile*, in which the author states: «And because architecture, as a discipline that uses measurements in all its operations, depends on geometry and must know at least its basic elements»[17]. It is also important to remember the systematic approach taken by the treatise writer in every field of science he deals with. One might then ask: how could Guarini have dealt with the subject of the measurement of buildings without addressing all the possibilities and varieties of surfaces and volumes, also in view of the critical developments in the study of Geometry in the 17th century?
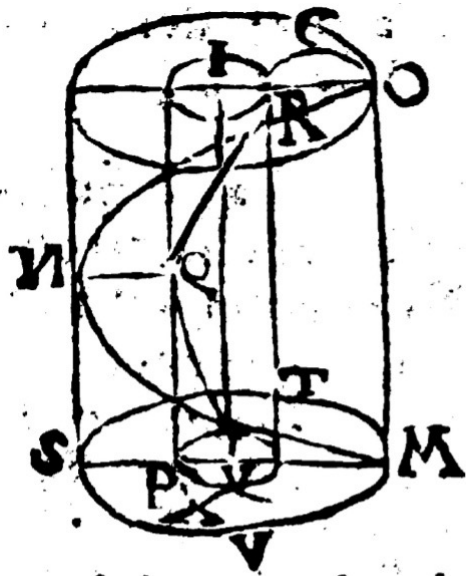
It is well known that the main problem in calculating surfaces and volumes in the period and cultural context in which the Theatine worked was related to complex vaulted systems in brick masonry.

He was one of the leading designers and experimenters of these vaults during the Baroque era. In Turin and the Piedmont area, he and several contemporary archi-

tects found fertile ground for inventing new configurations of considerable geometric complexity, built without intermediate pillars (fig. 5). In *Modo di misurare le fabriche*, the decomposition of these structures into simple elements, referable to 3D geometric surfaces, explores numerous possibilities, including those attributable to conical sections rediscovered a few decades earlier in Descartes' analytical geometry. The theme of calculating the intrados surface area of vaults, therefore, emerges as central to the second part of the volume. Among 36 propositions presented therein, 19 refer to them.

Compared to *Architettura civile*, where Guarini first systematised the typology of vaults[18] in geometric terms, the text examined here proposes over 30 types of shapes applicable to vaults, 20 of which add variations to the elementary cases presented in *Architettura civile*, bringing the examples closer to real cases. Among the most significant models are: the surface calculation of a lowered pavilion vault on a rhombus plan, discussed in Part Two, Proposition 12, which can be referred to the angular vault in the street-facing atrium of Palazzo Provana di Collegno (fig. 6), designed by Guarini, and the elliptical section groin, examined in Part Two, Proposition 23, present in the courtyard atrium of the same building (fig. 7), and, finally, the calculation, apparently unrelated to architectural references, of the surface area of a concave pyramid in Part Two, Proposition 24, which is actually very useful for calculating the surface area of pointed groins (fig. 8).

Explicit references to architecture, therefore, seem to focus on the most complex shapes to measure, which are also immediately reflected in Guarini's designs.

Perhaps it is no surprise that a further reference to architecture is linked to the calculation of the intrados surface area of spiral staircases, resolved in Part Two, Proposition 33, which is reflected in the two small oval twin staircases in Palazzo Carignano (fig. 9).
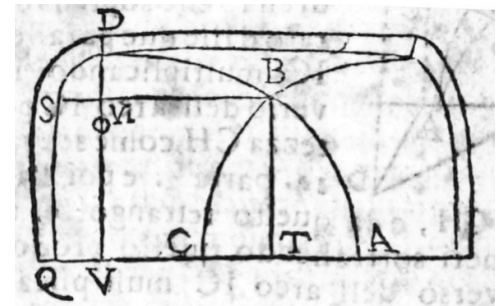
Only one proposition deals with survey techniques. Indeed, in Part two, Proposition 16 addresses a problem related to the lunette vaults: specifically, calculating the surface to be subtracted from the main surface, which consists of a quarter of a cylinder after cutting with vertical planes. The author explains the necessity of such a calculation to achieve a correct measurement of the lunette vaults: «As it is reasonable to measure the lunettes, which certainly make greater the surface of a vault, so it is convenient to remove that surface from the same vault, which occupies the space of the lunette»[19]. Guarini explains how to survey the position of specific points and the horizontal plane tangent to the vertex of the groin, finalising these operations with the calculation (fig. 10). In the empty space, a lunette must be inserted.

Conclusions

The analysis carried out highlights the need to read Guarini's writings on architecture as a complementary system of highly integrated knowledge. Guarini's position on surveying and measurement must therefore be analysed by combining the contributions of four treatises: *Euclides adauctus*, *Architettura civile*, *Trattato di fortificatione*, and *Modo di misurare le fabriche*.

In particular, the link between geometry and architecture, intrinsic to Guarini's treatises, is expressed in *Modo di misurare le fabriche* through the concept of measurement.

The work stands out for its absolute novelty in the treatise panorama: it refers, in fact, to the construction phase and not to the survey of ancient works, and deals with the calculation of surfaces and volumes, not with operations aimed at representing measurements referring to orthogonal views of the building and to scale. The 3D diagrams accompanying the work bear witness to this approach and are adequately drawn to aid understanding of the textual content.

9 | *Intrados surface area of spiral staircase. (From G. Guarini, Modo di misurare le fabriche 1674, p. 132). Intrados surface of the oval staircase in Palazzo Carignano by Guarini. Photo by F. Natta.*

10 | *Surface to be subtracted from the main vault for inserting a lunette. (From G. Guarini, Modo di misurare le fabriche 1674, p. 110).*

[17] Guarini, *Architettura civile*, p. 3.
[18] Spallone, Vitali, *Star-shaped and Planterian Vaults in the Baroque Atria of Turin*, pp. 91-117.
[19] Guarini, *Modo di misurare le fabriche*, p. 109.

Bibliography

M.G. Bevilacqua, R. Spallone, *Composed Graphic Scales in the European Military Treatises and Manuals from the 17th to the 19th Centuries*, in L. Hermida González, J.P. Xavier, I. Pernas Alonso, C. Losada Pérez, *Graphic Horizons. EGA*, Springer, Cham 2024, pp. 180-189.

C. Bianchini, *La scienza della rappresentazione nella concezione di Guarino Guarini*. Gangemi, Roma 2008.

J. Calvo-López, *Stereotomy*, Birkhäuser, Basel 2020.

M. Docci, D. Maestri, *Storia del rilevamento architettonico e urbano*, Laterza, Roma- Bari 1993.

G. Guarini, *Euclides adauctus et methodicus, matematicaque universalis*, Typis Bartolomaei Zapatae, Torino 1671.

G. Guarini, *Modo di misurare le fabriche*, Appresso gl'Heredi di Carlo Gianelli, Torino 1674.

G. Guarini, *Trattato di fortificatione, che hora si usa in Fiandra, Francia, et Italia; composto in ossequio del sereniss. principe Lodovico Giulio cavagliere di Savoia*. Appresso gl'Heredi di Carlo Gianelli, Torino 1676.

G. Guarini, *Architettura civile*, Appresso Gianfrancesco Mairesse, Torino 1737.

H.A. Meek, *Guarino Guarini and his architecture*, Yale University Press, New Haven and London 1988.

H.A. Millon, *La geometria nel linguaggio architettonico del Guarini, in Guarino Guarini e l'internazionalità del Barocco*, Accademia delle Scienze, Torino 1970, vol. 2, pp. 35-58.

C.S. Roero, *Guarino Guarini and Universal Mathematics,* in *Nexus Network Journal*, 2009, 11, pp. 415–439.

A. Scotti Tosini, *Testo e immagini nell'Architettura civile e nelle opere teoriche di Guarini*, in G. Dardanello, S. Klaiber, H. A. Millon, *Guarino Guarini*, Allemandi, Torino 2006, pp. 89-105.

R. Spallone, *Rappresentazione e progetto. La formalizzazione delle convenzioni del disegno architettonico*, Edizioni dell'Orso, Torino 2012.

R. Spallone, *Geometry, Arithmetic, Architecture. Calculation Methods for Vault Surfaces in the Modo di Misurare le Fabriche by Guarini*, in L. Cocchiarella, *ICGG 2018 - Proceedings of the 18th International Conference on Geometry and Graphics*, Birkhäuser-Springer, Cham, Switzerland 2019, pp. 2108-2119.

R. Spallone, *Geometry of vaulted systems in the treatises by Guarino Guarini*, in *Revista de Expresión Gráfica en la Edificación*, 2019, 11, pp. 74-88.

R. Spallone, *Lunette Vaults in Guarini's Work. Digital Models between Architettura Civile and Modo di Misurare le Fabriche*, in *diségno*, I, 2019, 4, pp. 91–102.

R. Spallone, M. Vitali, *Star-shaped and Planterian Vaults in the Baroque Atria of Turin / Volte stellari e Planteriane negli atri barocchi in Torino*, Aracne, Ariccia, pp. 85-148.

B. Tavassi La Greca, *La posizione del Guarini in rapporto alla cultura filosofica del tempo*, Id. (a cura di) G. Guarini, *Architettura Civile*, Il Polifilo, Milano 1968, pp. 439-459.

L. Vagnetti, *La teoria del rilevamento architettonico in Guarino Guarini*, in *Guarino Guarini e l'internazionalità del Barocco*, t. I, Accademia delle Scienze, Torino 1970, pp. 497-511.